Nuclear structure studies with low energy $\bar{p}$

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Abstract. The nuclear $\bar{p}$ capture from atomic states is briefly reviewed and several capture modes are compared. All these modes may test neutron density distributions in different regions of nuclei and yield complementary information on the $R_{\text{rms}}$ and higher moments of the neutron density profiles. Some advantages and difficulties of experimental methods are indicated and a special attention is paid to the $\pi^+$ emission following $\bar{p}$ annihilation. It is shown that this useful method may become very powerful if it determines more than the separate $\pi^+$ and $\pi^-$ multiplicities. Two specific questions are analysed: the ratio of $\bar{p}n$ and $\bar{p}p$ annihilation rates and the reabsorption of $\pi$ mesons in the residual nuclei.

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INTRODUCTION

About half century ago it was realized that kaonic and antiprotonic atoms provide a way to study the tail of the nuclear density distribution, its isospin structure and nuclear correlations there [1]. Three methods have been used so far, which give information in the regions roughly 1 fm to 3 fm beyond the half density radius.

- The observation of X-ray cascades in hadronic atoms and extraction of the atomic level shifts and widths [2, 3, 4, 5, 6, 7]. These level widths, related to the nuclear absorption of hadrons may determine high moments of the nuclear density distributions. Information obtained in this way is limited to one, in some special cases to two level widths and one level shift. The latter is in general difficult to interpret and provides a check on interaction models.

- Studies of the nuclear absorption products related to $\bar{p}$ annihilation

\[ \bar{p} + (N, Z) \rightarrow (N, Z - 1) + \text{mesons} \quad (1) \]

\[ \bar{p} + (N, Z) \rightarrow (N - 1, Z) + \text{mesons} \quad (2) \]

where a nucleus of $N$ neutrons and $Z$ protons is denoted by $(N, Z)$. Thus (1) involves predominantly the $\bar{p}p$ and (2) the $\bar{p}n$ annihilation. In this way one can discriminate captures on protons from captures on neutrons and obtain more nuclear information. However, these results are also more difficult to describe since the initial states of capture are not known directly and the final mesons may exchange charge or become absorbed. Some additional information is required.

- One way to study such reactions is the detection of mesons. This research was initiated in the nuclear emulsion with $K^-$ atoms [8, 9] and in bubble chambers with the $\bar{p}$ atoms [10, 11]. Scintillator detectors and fixed targets have later
been used [12]. Recently, a novel method to study the pion emission from trapped $\bar{p}$ atoms of unstable nuclei has been proposed [13]. Complementary measurements detect residual nuclei instead of the mesons. The radiochemical method was proven to be successful [14],[15]. In particular the final nuclei which may be detected in this way are "cold" Z-1, N-1 nuclei of excitation energies less than the neutron emission thresholds.

The basic $\bar{p}N$ interactions required for this research are limited to several phenomenological parameters: range of the $N\bar{N}$ annihilation, absorptive parts of the scattering amplitudes, pion production multiplicities, pion momentum distributions. These may be taken from other experiments while effects of uncertainties must be quantified.

**ATOMIC ANTIPROTONS**

Antiprotons bound into atomic orbits cascade down to annihilate within the nucleus. The annihilation happens at the extreme nuclear surface due to the two effects: the atomic cascade populates states of high centrifugal barrier and the free path of antiprotons in nuclear matter is less than 1 fm. The peripherality of capture allows for low density approximations: quasi-free scattering and single-particle picture of the nucleus. It simplifies the description of final mesons, an important point for understanding the absorption experiments. On the other hand the difficulty inherent in the surface studies is related to its sensitivity to $N\bar{N}$ force range.

The tool to describe the antiprotonic atomic level shifts and widths is an optical potential $V^{opt}$. The simplest one is assumed usually [4, 5, 6, 7, 16] in the form

$$V^{opt}(R) = \frac{2\pi}{\mu_{N\bar{N}}}\Sigma t^s_{N\bar{N}}\rho^s(R)$$

where $\mu_{N\bar{N}}$ is the $N\bar{N}$ reduced mass, $\rho^s(R)$ are nuclear densities at a radius $R$, index $s$ denotes protons or neutrons. The $t^s_{N\bar{N}}$ are complex "effective" scattering lengths. The finite range in the $N\bar{N}$ interaction requires folded densities

$$\rho^s(R) = \int d\mathbf{u}\rho^s_0(R-\mathbf{u})v(\mathbf{u})$$

instead of "bare" nucleon densities given in terms of single nucleon wave functions $\psi^G_N$, as

$$\rho^s_0(R) = \Sigma_\alpha[\psi^G_N(R)]^2$$

The form-factor $v(\mathbf{u})$ represents $N\bar{N}$ force range $r_\alpha$. For the absorptive part of $V^{opt}$, the annihilation range of 1 fm is expected from models of $N\bar{N}$ annihilation [24], and nuclear $\bar{p}$ scattering experiments [18]. The potential itself is highly absorptive $\text{Im} t^s_{N\bar{N}} \approx 2$ fm and at the nuclear centre $\text{Im} V^{opt}$ would be 200 MeV strong. The corresponding free path length is well below 1 fm. However, it should be remembered that the form and the strength of $V^{opt}$ is tested only in the surface region. In particular, $\text{Im} V^{opt}$ is determined
by the atomic level widths via
\[
\frac{\Gamma}{2} = \frac{2\pi}{\mu_{NN}} \sum_s \text{Im} t_{NN}^s \int dR \rho^s(R) \left| \Psi_{NN}(R) \right|^2
\]
(6)

where \( \Psi_{NN}(R) \) is the atomic wave function. Since \( \Psi_{NN} \approx R^l \) and only high angular momenta \( l \) are available, the absorption strength is peaked at the surface. In a crude approximation, the atomic widths are determined by \( <R^{2l}> \)-th moments of the nuclear density. Precise atomic wave functions, corrected for nuclear interactions, indicate the dominant moment to be \( <R^{2l-2}> \).

The form of optical potential is not fully settled, in particular the relation of the effective amplitude \( t_{NN}^s \) to the low energy expansion parameters
\[
t_{NN} = a_N + 3b_N p p'
\]
(7)
where \( a \) are scattering lengths, \( b \) are scattering volumes and \( p \) are momenta. Such an amplitude generates the optical potential composed of central and gradient terms
\[
V_{opt} = \sum_s V_s = \frac{2\pi}{\mu_{NN}} \sum_s \left[ a_s \rho^s + 3b_s \nabla \rho^s \nabla \right]
\]
(8)

Overall, the X ray data indicates \( t_{NN}^{protons} \approx t_{NN}^{neutrons} \), [20], but other experiments show more complicated structure. These questions are discussed later in the text.

**STUDIES OF ANTIPROTON ANNIHILATION PRODUCTS**

In this section we discuss nuclear reactions induced by the annihilation. The point of special interest is the effect of the final state mesons upon the formation of final states detected in specific experimental conditions.

The partial widths \( \Gamma_s \) for reactions (1,2) leading to a final state \( s \) may be described by a formula analogous to eq. (6)
\[
\frac{\Gamma_s}{2} = \frac{2\pi}{\mu_{NN}} \text{Im} t_{NN}^s \int dR \left| \Psi_{NN}(R) \right|^2 \rho_s(R) P^s(R)
\]
(9)

where some functions \( P^s \) are introduced in order to describe the formation of required final states. For an isolated \( NN \) system or in case of full atomic level width eq.(9), summed over all states \( s \), reduces to the unitarity condition for the absorptive part of the elastic scattering amplitude \( \text{Im} t_{NN}^s \). Inside a nucleus this condition is modified by the meson interactions and the selection of final states. A reliable description of \( \Gamma_s \) is facilitated by large multiplicity of mesons (\( \approx 5 \)) and large (\( \approx 2 \text{ GeV} \)) deposit of the annihilation energy. On the nuclear scale, and on the scale of each individual meson, the closure approximation is justified, and it leads to the simple formula (9). The calculation of \( P^s \) is now briefly outlined. Finer details: finite range of interactions, meson multiplicities and correlations, unitarity, applicability of the closure approximation and other corrections are described at some length in ref. [22].
Let the antiproton in a state denoted by \( n \) and described by a wave function \( \Psi_N^\bar{n}(R) \) annihilate on a nucleon in a single particle state \( \alpha \) of wave function \( \varphi_N(R)^\alpha \). The outcome is \( k \) pions with momenta \( p_i \) and other quantum numbers denoted jointly by \( \xi_i \), \( i = 1...k \). The transition amplitude for this process, is

\[
f_{n,\alpha} = \int \Psi_N^\bar{n}(R) \varphi_N(R)^\alpha t_{NN\rightarrow M}(\xi) \prod_i \tilde{\varphi}_M(R, \xi_i),
\]

where \( t_{NN\rightarrow M} \) is a transition matrix that describes the annihilation. All wave functions in eq. (10) involve nuclear interactions. In the bulk of the phase space, pions are fast enough to allow an eikonal description. Following this, the wave function \( \varphi_M \) for each pion is taken in the form of ingoing waves

\[
\tilde{\varphi}_M(R, \xi_i) = \exp(i p R - i S_i(p, R))
\]

with \( S \) calculated in terms of a pion-nucleus potential \( U_i \)

\[
S_i(p, R) = \int_0^\infty dt [\sqrt{(p^2 - U_i(R + \hat{p}t) - p)}]
\]

by integrating the local momentum over a linear meson trajectory. Due to nuclear excitations and pion absorptions this wave is damped with a rate described by Im\( \delta \).

To calculate the absorption probability, amplitudes (10) are squared, summed over the final pionic channels and integrated over the phase space. The latter includes also the total momentum of the pions \( P = \sum p_i \) equal to the recoil momentum of the residual nucleus. The nuclear recoil and excitation energy is negligible with respect to the total energy release and the closure approximation is applicable. That simplifies the mesonic part of the bilinear expression \( f\bar{f} \) summed over pionic states

\[
\sum_k \int dP \int d\xi t_{NN\rightarrow M}^s(\xi) t_{NN\rightarrow M}^s(\xi) \varphi_M(R, \xi) \tilde{\varphi}_M(R', \xi) \approx
\]

\[
\text{Im} t_{NN}^s \delta(R - R') < \prod_i \left| \exp(-S(p_i, R)) \right|^2
\]

(13)

Here, \( d\xi \) denotes integrations over the relative pion momenta restricted by the energy conservation. The integration over momentum \( P \) generates (approximately) \( \delta(R - R') \) and the sum covers pion multiplicities. For an isolated \( N\bar{N} \) system eq. (13) reduces to the unitarity condition for the absorptive part of the elastic scattering amplitude Im\( t_{NN}^s \). Inside nuclei this condition is modified by a product of functions

\[
P_{miss}(R) = \left< \prod_i \left| \exp(-S(p_i, R)) \right|^2 \right>
\]

(14)

which describe final state interactions of each meson, averaged over the multiplicities and phase space.

In the case of radiochemical measurements the summation and averaging is done over all final mesons. In addition there are some limitations on the final nuclear states detected in the experiment. That leads to two final state functions describing different physical
effects, $P^s = P_{\text{miss}}(R)P_{\text{dh}}(R)$. The chance that all mesons miss the residual nucleus is given by the $P_{\text{miss}}$ of eq. (14) which describes final state interactions of mesons. The closure approximation materialized in eq. (13) allows the transition from eq. (6) to eq. (9) in the text, as the squares of $q^\alpha_N$ reduce to the nuclear density. Most of the final meson interactions remove the residual nucleus from the "cold" $A-1$ state. Thus $S$ is generated by $U^{opt}$ which includes all final meson interactions: the true meson absorption and the scattering which is predominantly an inelastic scattering [30]. Under these conditions the optical potential $U^{opt}$ is determined mainly by the absorptive $\pi - N$ scattering amplitude in the $\Delta$ and higher resonance region. The other factor $P_{\text{dh}}$ represents a correction for the $\bar{p}$ annihilation on "deeply bound" nucleons. A fast annihilation of a nucleon leaves the nucleus with a hole in one of nuclear shells. Later, the nucleus rearranges into an excited system. If the related excitation energy exceeds $\approx 8$ MeV a neutron is rapidly emitted. Such annihilations lead to "hot" $A-1$ nuclei, and the true residuals become $A-2$ nuclei. One should remove those "deep hole" contributions, and a good estimate for $P_{\text{dh}}$ is

$$P_{\text{dh}} = \Sigma_{\alpha, \text{upper}}[q^\alpha_N(R)]^2/\Sigma_{\beta, \text{all}}[q^\beta_N(R)]^2$$

(15)

where the first sum is limited, by the value of the neutron emission threshold, to the upper nucleon orbitals. At the surface this correction is small, but it becomes relevant for more central absorptions.

In experiments that detect a final meson of charge $s$ the states of residual nuclei are irrelevant. Hence, $P_{\text{dh}} = 1$ and

$$P^s(R) = \langle |\exp(-S_s(p, R))|^2 \rangle \equiv P^s$$

(16)

describes the chance of survival for each charged meson. The relevant calculation should involve only true $\pi^+, \pi^-$ absorptions as well as $\pi^+ \rightarrow \pi^0$, $\pi^- \rightarrow \pi^0$ charge exchange reactions. From the scattering cross sections, the absorptive and charge exchange one [30], the first process is expected to dominate as 4:1. The averaging in eq. (16) involves meson multiplicities and the meson phase space. Some results are discussed in the last section.

The localisation of antiproton capture events is given by a product of the densities and the final state factors $|\Psi_s(R)|^2 \rho_s(R)P_s(R)R^2$. The relevant regions are indicated in fig.1. The X-ray measurements offer an advantage of well defined initial and final states, but cannot distinguish the proton contribution from the neutron one. The other methods can do that, but the knowledge of initial $\bar{p}$ states is limited. In principle the composition of capture states may be obtained in cascade calculations, but additional data are required to check these. In the cold captures the necessary constraints are given by the ratios $\Gamma(A-1)/\Gamma(\text{total})$ which amount to $\approx 10\%$. In the meson emission studies, equivalent constraints are given by the meson absorption rate $P_\pi$. Both these quantities are obtained in the relevant experiments. Figure 1 shows that the three discussed methods offer complementary information on different regions of the nuclear densities. The X-ray data involve densities of $\approx 5\%$ of the central one around $c + 1$ fm, where $c$ is the half density radius. The mesonic measurements are slightly more peripheral while the radiochemical method tests the most extreme region. These differences reflect the
TABLE 1. Brief comparison of the three methods using antiprotons to study the neutron skin.

<table>
<thead>
<tr>
<th>Method</th>
<th>measured quantity</th>
<th>advantages</th>
<th>difficulties</th>
</tr>
</thead>
<tbody>
<tr>
<td>X-rays</td>
<td>level widths $\Gamma$</td>
<td>$\bar{p}$ state known</td>
<td>no $p,n$ separation</td>
</tr>
<tr>
<td></td>
<td>level shifts $\rightarrow$</td>
<td>check on $V_{\bar{p}}^{optical}$</td>
<td></td>
</tr>
<tr>
<td>Cold capture</td>
<td>$\Gamma(\bar{p}n)/\Gamma(\bar{p}p)$</td>
<td>$n,p$ separation</td>
<td>$\bar{p}$ state unknown</td>
</tr>
<tr>
<td></td>
<td>$\Gamma(A-1)/\Gamma(total)$</td>
<td>check on $\bar{p}$ states</td>
<td></td>
</tr>
<tr>
<td>Meson emission</td>
<td>$N(\pi^-) - N(\pi^+)$</td>
<td>$n,p$ separation</td>
<td>$\bar{p}$ state unknown</td>
</tr>
<tr>
<td></td>
<td>$N(\pi^-) + N(\pi^+)$</td>
<td>check on $\pi$ absorption</td>
<td></td>
</tr>
</tbody>
</table>

FIGURE 1. A schematic representation of the antiproton capture regions in $^{112}$Sn that could be tested in three atomic experiments: X-rays, cold A-1 capture Warsaw – Munich and charge meson detection, Riken. AIC - denotes the in flight capture proposal [17].

chances for mesons to leave the nucleus. The $P^v(R)$ are very small at central densities and rise only in the nuclear surface region. All characteristic features of the three methods are briefly summarised in table 1.

All methods require additional data: the strengths of the capture rates $\sigma(\bar{p}n)$ and $\alpha(\bar{p}p)$ or their ratio $R_{n/p} = \sigma(\bar{p}n)/\sigma(\bar{p}p)$. The experiments which measure the capture rates on neutrons $\Gamma(\bar{p}n)$ and protons $\Gamma(\bar{p}p)$ determine a "halo factor" [10] defined as

$$
\frac{\Gamma(\bar{p}n)}{\Gamma(\bar{p}p)} = \frac{Z}{N}R_{n/p}^{\text{halo}}.
$$

(17)

With the $R_{n/p}$ known, $^{\text{halo}}$ represents the neutron excess in the capture region. It is left to the nuclear theory to disclose what region of the nuclear surface is involved and what
is the physical origin of the halo. In the next sections the $R_{n/p}$ is extracted from several experiments and the capture site in the mesonic measurements is discussed.

**The $\sigma(\bar{p}n)/\sigma(\bar{p}p)$ ratio**

The $R_{n/p}$ may be extracted from the low energy in-flight captures, $\bar{N} - N$ interaction models and the antiproton optical potential fitted to the X-ray data.

The atomic widths for partial absorptions $\Gamma(\bar{p}n), \Gamma(\bar{p}p)$ are related to the absorptive amplitudes $\text{Im} \ t_{NN}$ for the $\bar{p}n$ and $\bar{p}p$ pairs. In the $\bar{NN}$ scattering states these amplitudes are given by the total cross sections $\sigma(\bar{p}n), \sigma(\bar{p}p)$. For bound nucleons such relations are not possible since there are no cross sections. One has to extrapolate the amplitudes to negative energies and check the results against other available experiments. The amplitudes determined in this way allow to calculate the antiproton optical potentials. Next, these potentials could be used in the analysis of the nuclear capture data.

The number $R_{n/p}$ required in the neutron halo studies has been obtained in several experiments listed in table 2. The numbers seem random but, as shown below, there is some simplicity behind these results. Let us notice that the $\bar{p}N$ scattering amplitudes $t$ describing the scattering on a bound nucleon enter the formalism as $t(E_{\bar{p}} - E_B - E_{\text{recoil}})$, where $E_B$ is the nucleon binding and $E_{\text{recoil}}$ is the recoil energy of the $\bar{p}N$ pair with respect to the residual system. In atomic states the energies $E_{\bar{p}}$ are slightly negative while in the "stopped" antiproton states $E_{\bar{p}} \approx 1$ MeV, [29]. The tested quantity is the scattering amplitude

$$\tilde{t} = \int t(E_{\bar{p}} - E_B - \frac{p^2}{2m_R}) |\tilde{\phi}(p)|^2 d\tilde{p}$$

(18)

weighted by the recoil momentum distribution. The latter is determined by $\phi(p)$, the Fourier transforms of $\Psi_{\bar{p}N}^n(R) \psi_{NN}^n(R)$. Typical energies involved in eq.(18) are (-2,-8) MeV in D, (-8,-40) MeV in heavy nuclei, and (-20,-40) MeV in $^4$He. These energies cover the unphysical subthreshold region and a model is required for such an extrapolation. Here, the amplitudes averaged over spin states are calculated in terms of Paris potential [24]. This potential has been fitted to the 3400 $\bar{p}p$ and $\bar{n}n$ scattering data and the recent $\bar{n}p$ scattering measurements ref. [25] are essential for our purpose.

There are four basic antiproton amplitudes of interest in eq. (7). Figure 2 gives $R_{n/p} = \text{Im} a_n/\text{Im} a_p$ calculated for the S wave and compared to the data. Similar calculations for P waves can be compared only to the atomic data as shown in fig. 3. The following procedure was adopted to obtain these results.

1. The stopped $\bar{p}$ experiments involve essentially the S wave capture. For He targets the initial experimental momenta of 10 MeV/c up to 50 MeV/c [29] indicate pure S wave capture. On the other hand, the deuteron data [28] may contain some higher momenta and a P wave component. In the figure, the experimental ratio is compared to the ratio calculated at the threshold. The calculated $R_{n/p}$ compare well with the chamber data indicated in the figure. For P waves the calculated $R_{n/p} \approx 1$. 

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TABLE 2. The experimental antiproton capture ratios $R_{\bar{n}/p} = \sigma(\bar{n}p)/\sigma(\bar{p}p)$ extracted in atomic states and in states of flight. In C and Ni nuclei, the relation $\rho_p/Z = \rho_n/N$ was assumed.

<table>
<thead>
<tr>
<th>Element</th>
<th>$R_{\bar{n}/p}$</th>
<th>method</th>
<th>state</th>
<th>reference</th>
<th>remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>0.81(3)</td>
<td>chamber stopped</td>
<td>[28]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^3$He</td>
<td>0.47(4)</td>
<td>chamber stopped</td>
<td>[29]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^4$He</td>
<td>0.48(3)</td>
<td>chamber stopped</td>
<td>[29]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{12}$C</td>
<td>0.63</td>
<td>$\pi^+,\pi^-$ atoms</td>
<td>[10]</td>
<td>$\rho_p = \rho_n$</td>
<td></td>
</tr>
<tr>
<td>$^{58}$Ni</td>
<td>0.8</td>
<td>cold A-1 atoms</td>
<td>[22]</td>
<td>$\rho_p = \rho_n$</td>
<td></td>
</tr>
<tr>
<td>Z=8-90</td>
<td>$\approx 1$</td>
<td>X-rays atoms</td>
<td>[20]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Z=8-90</td>
<td>$\leq 1$</td>
<td>X-rays atoms</td>
<td>[21]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Z=50-90</td>
<td>$\approx 1$</td>
<td>X ,cold A-1 atoms</td>
<td>[19]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

FIGURE 2. The $R_{\bar{n}/p}$ ratio obtained in low energy in-flight capture experiments, [29, 28]. The horizontal scale indicates the energies involved in the $\bar{n}N$ CM systems, relevant to these experiments. The two curves correspond to the Paris potential calculation of the ratio in S and P wave $\bar{n}N$ interactions.

(2) In the case of atoms the $R_{\bar{n}/p}$ involves mixtures of the S and P wave interactions. Thus, it depends on the state and cannot be presented as a single curve. A good guidance is given in terms of the optical potential for antiprotons $V^{opt}$ of eq. (8). The strengths of $\bar{n}n$ and $\bar{p}p$ capture rates are determined by average values of $V^{opt}$ in atomic states characterized by the principal quantum number $n$ and angular momentum $l$. The ratio becomes

$$R_{\bar{n}/p} = < n, l | ImV_n | n, l > / < n, l | ImV_p | n, l > .$$  \hspace{1cm} (19)

Roughly one has

$$< n, l | V_s | n, l > \sim < \rho^s(R)[a_s + 3b_s \frac{l(2l+1)}{R^2}] >_{n,l}$$  \hspace{1cm} (20)

where R is the radial coordinate. The contribution from the P wave term $b$ dominates and its role increases with the increasing atomic number. That happens for the orbitals tested.
in the X-ray cascade since the average ratio of the antiproton angular momentum to the annihilation distance $<l^2/R^2>$ increases slowly with the nuclear size. With the values of $a, b$ obtained in the Paris model one obtains $R_{n/p} \approx 2/3$ in Carbon, and this number increases slowly to $R_{n/p} \approx 1$ in heavy nuclei. The trend is indicated in fig. 3 but the real numbers must be calculated specifically for each state of interest. The trend compares well with the results given in table 1.

(3) An optical potential [21] indicates a fair consistency with the $\bar{N}N$ model calculations. In particular the best fit parameters extracted from the X-ray data $\text{Im} a = -0.85$ fm, $\text{Im} b = -0.80$ fm$^3$ are fairly close to the Paris model averages calculated at a characteristic subthreshold energy of -20 MeV. These are not final results, both the optical potentials and the $\bar{N}N$ potential models are being improved. In particular one needs to understand the important deuteron and helium atoms as well as the real parts of $a$ and $b$.

One has reached an almost quantitative agreement of the elementary $\bar{N}N$ model based upon the $\bar{N}N$ data and the nuclear $\bar{p}$ capture experiments. It gives a good prospect for a reliable calculations of $R_{n/p}$.

**THE ANALYSIS OF $\pi^+$ AND $\pi^-$ MULTIPLICITIES**

This section is devoted to the analysis of the RIKEN proposal [13] and its potential advantages. The analyses of the mesonic measurements, aimed to extract the halo factor, consist of two steps [10, 11]. First, one finds the pion emission probability $P_\pi$ given by the total number of the emitted pions $N(\pi^-) + N(\pi^+)$ and the total capture events...
number $N_{\text{events}}$. These define the charged pion multiplicity $M(\pi^\pm)$

$$M(\pi^\pm) = \frac{N(\pi^-) + N(\pi^+)}{N_{\text{events}}} = M_{\text{prim}}(\pi^\pm)P_\pi$$  \hspace{1cm} (21)

in terms of the pion emission probability $P_\pi$ and a primary multiplicity $M_{\text{prim}}(\pi^\pm)$. The latter is taken from the deuteron. The second step is to determine $N(\bar{p}n)$, the number of events due to $\bar{p}n$ captures

$$N(\bar{p}n) = \frac{N(\pi^-) - N(\pi^+)}{P_\pi}. \hspace{1cm} (22)$$

Next, one obtains $N(\bar{p}p) = N_{\text{events}} - N(\bar{p}n)$ and the halo factor follows from eq. (17) [$N \sim \Gamma$]. In those analyses it has been assumed that $N(\pi^-) \pm N(\pi^+)$ are not much affected by the absorption of several mesons and by the charge exchange $\pi^0 \to \pi^+$, $\pi^0 \to \pi^-$ in the final states. These corrections will be studied later, let us first look into the consequences of Eqs.(21,22).

The early experiments yield $M(\pi^\pm)$ in 5 elements C, N, Ti, Ta, Pb [10, 11]. Later, more extensive measurements found $M(\pi^\pm)$ for 15 elements, albeit with no $\pi^+$ and $\pi^-$ distinction [12]. These measurements and the related emission probabilities allow to locate the $\bar{p}$ annihilation site. Calculations of ref. [23] by the way of eq. (16) prove a simple, but quite precise, geometric interpretation of $P_\pi$ in terms of a solid angle $\Omega$ under which the nucleus is seen from the annihilation point. A simple geometry yields

$$P_\pi = 1 - \frac{\Omega}{4\pi}, \quad \frac{\Omega}{2\pi} = 1 - \sqrt{\frac{1}{c/(c + \delta)}}^2 \hspace{1cm} (23)$$

where $\delta$ is the radial distance from the nuclear half density radius $c$ to the average capture radius $c + \delta$. Two factors generate such a simple picture. First the $\pi$ absorption is very strong and second it involves two (or more nucleons). The related $\rho^2$ absorption profile narrows the surface region and the pions see the nucleus as a black sphere. The best fit to the available $P_\pi$ and the nuclear pion absorption cross sections [30] produces $\delta = 1.35(0.1)$ fm [23]. The real absorption region has some 2 fm radial depth centered at $c + \delta$ as given by eq. (9) and indicated in figure 1.

However, such analyses have not used the full power of the experiment. In particular the bubble chamber measurements produce 7-8 multiplicities for the total mesonic charges. Two examples are given in table 3. To understand these charge multiplicities a simple phenomenological calculation is now performed. Let the primary meson multiplicities be $W[n,k]$ where $n$ is the total number of mesons and $k$ is the number of charged $\pi^+, \pi^-$ pairs. The multiplicities $W$ are well known from $\bar{p}$ stopped in hydrogen and deuterium followed by the charged pion emissions [26]. The inclusion of $\pi^0$ may be done fairly precisely with the inclusion of single $\pi^0$ production data, total meson production data and the statistical model [27].

Consider the process initiated by a primary $\bar{p}p$ annihilation in a nucleus. The observed final multiplicities differ from the primary multiplicities because of two effects: meson absorptions and meson charge exchanges. Let us discuss the absorption first. and denote
TABLE 3. The experimental and calculated in a phenomenological way pion charge multiplicities following the \( \bar{p} \) absorption in C and Pb nuclei. The unit is %. The C nucleus: the data of ref. [11] are used with the errors calculated in ref. [26]. The other data of Bugg [10] were corrected for the hydrogen contamination by a simple subtraction of the hydrogen events in the charge 0 sector. No errors are attributed to these results. The parameters used in this calculation \( R_{n/p} = 0.66, P_x = 0.13 \) \( \omega = 0.2, \lambda = 0.1, \text{f}_{\text{halo}} = 1 \) should be compared to \( R_{n/p} = 0.63(6), P_x = 0.13(1), \text{f}_{\text{halo}} = 1 \) from ref. [11]. The Pb nucleus: data from ref. [10], with no available experimental errors. The parameters used in this calculation \( R_{n/p} = 0.96, P_x = 0.235, \text{f}_{\text{halo}} = 1.8 \). The charge symmetric version c.s with \( \omega = 0.29, \lambda = 0.12 \) misses the charge (+1) and (-2) sectors. The charge non symmetric results n.c.s. with \( \omega_+ = 0.26, \omega_- = 0.32, \lambda = 0.12 \) are given in the last column. These numbers should be compared to \( R_{n/p} = 0.63(6), P_x = 0.221(0.014), \text{f}_{\text{halo}} = 2.34(50) \) from ref. [10].

<table>
<thead>
<tr>
<th>charge</th>
<th>exp.</th>
<th>calc</th>
<th>exp</th>
<th>exp</th>
<th>calc c.s.</th>
<th>calc n.c.s.</th>
</tr>
</thead>
<tbody>
<tr>
<td>- 2</td>
<td>1.8(2)</td>
<td>1.3</td>
<td>2.1</td>
<td>2.6(1)</td>
<td>1.3</td>
<td>1.7</td>
</tr>
<tr>
<td>+ 1</td>
<td>12.5(4)</td>
<td>13.0</td>
<td>17.5</td>
<td>15.1(1)</td>
<td>11.6</td>
<td>14.3</td>
</tr>
<tr>
<td>+ 0</td>
<td>43.0(8)</td>
<td>43.1</td>
<td>38.3</td>
<td>30.6(1)</td>
<td>30.5</td>
<td>32.3</td>
</tr>
<tr>
<td>- 1</td>
<td>34.5(7)</td>
<td>34.1</td>
<td>33.7</td>
<td>37.7(1)</td>
<td>39.2</td>
<td>37.0</td>
</tr>
<tr>
<td>- 2</td>
<td>6.5(3)</td>
<td>8.0</td>
<td>7.8</td>
<td>12.3(1)</td>
<td>16.2</td>
<td>13.8</td>
</tr>
<tr>
<td>- 3</td>
<td>1.0(1)</td>
<td>0.4</td>
<td>0.6</td>
<td>1.7(1)</td>
<td>1.0</td>
<td>0.8</td>
</tr>
</tbody>
</table>

the probabilities for a charged meson absorption by \( \omega_+ \) and \( \omega_- \). The probability that all charged mesons are emitted and the total charge 0 is observed equals to

\[
P[0] = \Sigma_{n,k} W[n,k](1 - \omega_-)^k(1 - \omega_+)^k,
\]

the probability that one \( \pi^- \) is absorbed giving total meson charge 1

\[
P[1] = \Sigma_{n,k} W[n,k](1 - \omega_-)^{k-1}\omega_- (1 - \omega_+)^k,
\]

in a similar way a double \( \pi^- \) absorption gives

\[
P[2] = \Sigma_{n,k} W[n,k](1 - \omega_-)^{k-2}\omega_-^2 (1 - \omega_+)^k
\]

and so on. Similar analysis may be done for the primary \( \bar{p}n \) annihilations but it turns out that the final multiplicities are not well reproduced. The charge exchange processes \( \pi^0 \to \pi^+ \) and \( \pi^0 \to \pi^- \) must be taken into account. Let the probability for \( \pi^0 \to \pi^\pm \) be \( \lambda_{\pi^\pm} \). With these processes the primary \( \bar{p}p \) annihilation contributes to the final charge 1 state

\[
\Delta P[1] = \Sigma_{n,k} W[n,k]P[n,k](1 - \omega_-)^k\lambda_+(1 - \omega_+)^k
\]

where \( P[n,k] \) is the fraction of \( \pi^0 \) in the \( n,k \) states [27]. The expression given above assumes also that all the charged pions are identified. With our phenomenological approach the inverse processes \( \pi^+ \to \pi^0 \) constitutes some fraction of the \( \pi^+ \) absorption.
probability already included in the parameter $\omega_+$. This procedure is continued and parameter $\lambda$ is extracted from the data. The results are shown in table 3. For C nucleus a charge symmetric solution with two parameters gives good fit to the data of ref. [11]. In this fit the overall absorption parameter $P_\pi$ was fixed to the experimental result given by eq. (21). The charge exchange parameter $\lambda$ turns out sizable. It reflects the effect of $\Delta (1233)$ resonance which produces charge exchange cross section of 30mb strength. The outcome in terms of $R_{n/p}$ is essentially the same as in the standard approach. This happens since the two uncertain processes $\pi^+ \rightarrow \pi^0$ and $\pi^0 \rightarrow \pi^+$ balance each other. The essential fact is that the measurements of the pion multiplicities allow a direct experimental control over both the absorption and the charge exchange processes. Let us also comment that the results from hydrogen chamber [10] which require uncertain background subtraction, offer much worse comparison with this phenomenological calculation. This underlines the need to remove the $\bar{p}p$ background.

Similar analysis made for Pb nucleus indicates that in the neutron excess situation the absorption of final $\pi^-$ is larger than that of $\pi^+$. As seen from table 3 the effect is small but noticeable. It is, yet again, related to the neutron halo but it is not a trivial one. The explanation may be given in terms of the $\pi NN \rightarrow \Delta N \rightarrow NN$ absorption model. The dominant mode involves $np$ pair with the deuteron quantum numbers. Thus, the pion absorptive cross section is not much affected by the neutron excess [30]. However, the geometry of the surface capture of antiprotons makes the difference. Thus, the $\pi^-$ meson born at distant surface is much more likely to collide with a neutron than with the proton. With the $\pi^- n$ collision the $\Delta$ resonance is more likely to be formed. Next, the $\Delta$ propagates by about 1 fm and interacts with a more central proton by a long range pion exchange force. This sizable range in the absorption mechanism, indicated in ref. [31], generates the asymmetry in the $\pi^-$ and $\pi^+$ behavior. An estimate of the propagation ranges and the geometry involved makes the ratio $\omega_- / \omega_+$ to rise up to about 4/3. Similar observations are also made in other (Ti, Ta) targets studied in the W.Bugg’s experiment.

Finally let us notice that such an extended analysis does not change the $f_{halo}$ results given by Bugg, within the error limit given by the standard method of eqs. (21) and (22). Some reduction obtained in the Pb case is due to the updated value of the $R_{n/p}$.

**CONCLUSIONS**

There are several methods which use antiprotons to study the neutron distribution in nuclei. These methods are complementary as they provide information on different nuclear regions. The pion emission experiments, which are of our special interest, may be a very useful tool to test the densities at nuclear radii about 1 fm beyond the nuclear half density radius. In particular:

- The pion emission measurements may provide not only the "halo factors" but also probabilities of the final pion absorption and charge exchange.
- The parallel theoretical calculations for the pion absorption and charge exchange probabilities will pinpoint the properties the initial $\bar{p}$ capture state.
- The essential primary rates of captures on neutrons and protons and their ratio $R_{n/p}$ are now fairly well understood as an interplay of the S and P wave absorptions. The S
wave is tested in the low energy absorption in flight, the interplay of S and P is tested in the antiprotonic X-rays.

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REFERENCES