XXI ICTAM, 15-21 August 2004, Warsaw, Poland

NONLINEAR OVERALL VISCOELASTIC PROPERTIES OF THE RANDOM MULTICOMPONENT MEDIA

Boris P. Maslov
National Academy of Sciences, S.P. Timoshenko Institute of Mechanics, Nesterova 3, 03047 Kyiv, Ukraine

Summary This work is concerned with the prediction of the effective or overall response of a random multi-component media with nonlinear visco-elastic constituents. The continuum considered here is suggested to be subjected to finite deformation. Kirchhoff stress tensor and deformation gradients are used as field variables in a fixed reference state. Nonlinear problem is investigated in second order approximation theory when the gradient deformation terms higher than second order are neglected. Five constant elastic potential in elastic problem and five time functionals in visco-elastic one are used to build overall constitutive relations.

Theoretical determination of the overall properties of inhomogeneous media is a problem of a great importance. This work is concerned with the prediction of the effective or overall response of a random multi-component media with nonlinear visco-elastic constituents. The elastic properties of inhomogeneous structures have been studied intensively during last decades. The works of R.M. Christensen, J.R. Willis, Z. Hashin and others represent a comprehensive review of achievements in this area. But there are a lot of questions arise in the case when visco-elastic response is a subject of investigation, nonlinear especially.

Certain representative meso-domain B containing a set of inclusions in reference configuration κ is considered. So the volume v_R of the body may be represented as

$$v_R = v_m \bigcup v_a, \qquad v_a = \bigcup_{i=1}^{n} v_i;$$

Each volume of v_r, $r \in [1, n+1]$, contains a visco-elastic material, the effective properties of which are determined by the potential of type

$$\hat{W}(\boldsymbol{E}, t) = \mu\left[\left(1 + \frac{1}{2}\alpha\right)I_1^2 - 2I_2 + \beta_1 I_1^3 + \beta_3 I_1 I_2 + \beta_3 I_3 + \mathrm{O}\left(\left|\boldsymbol{E}\right|^4\right)\right], \tag{1}$$

where I_i are the main invariants of Lagrange strain tensors

$$I_1 = tr(\boldsymbol{E}); \qquad I_2 = \frac{1}{2}\left(I_1^2 - tr\boldsymbol{E}^2\right);$$

$$I_3 = \frac{1}{3}\left(tr\boldsymbol{E}^3 - I_1^3 - 3I_1 I_2\right); \qquad \boldsymbol{E} = \frac{1}{2}(\boldsymbol{F}^T\boldsymbol{F} - \boldsymbol{1});$$

$$\boldsymbol{F} = \partial \boldsymbol{x}^A / \partial \boldsymbol{x} = \boldsymbol{1} + \boldsymbol{H}; \qquad \boldsymbol{x}^A = \boldsymbol{x} + \boldsymbol{u}(\boldsymbol{x});$$

$$\boldsymbol{H} = \partial \boldsymbol{u} / \partial \boldsymbol{x} = \nabla \boldsymbol{u}; \qquad \alpha = \lambda / \mu,$$

$\lambda, \mu, \beta_1, \beta_2, \beta_3$ stand for the integral convolution type operators of visco-elasticity or for constants of material in pure elastic case, tr - trace operator of appropriate tensor, $\boldsymbol{F}(t)$- the strain gradient, $\boldsymbol{H}(t)$- the gradient of displacement vector $\boldsymbol{u}(\boldsymbol{x}, t)$ taken in the coordinate system of the reference configuration, $\boldsymbol{1}$ - unit symmetric tensor of the second order.

So the corresponding Piola-Kirchhoff stress matrices at time t are given by

$$\sigma(t) = \mathop{\Phi}_{\tau = t_0}^{t}\{\boldsymbol{H}(\tau)\}, \tag{2}$$

where Φ is linear in time functional of hereditary type. The linear visco-elasticity with finite strain is therefore implemented here. Nonsymmetrical Piola-Kirchhoff first stress tensor σ and the gradient $\boldsymbol{H}$ of displacement vector $\boldsymbol{V}$ after being averaged over unstrained representative volume v_R of a composite B can be well used in terms of conjugated pair of macroscopic variables [1-3,5] as in nonlinear strain theory. Followed, the constitutive equation of non-homogeneous multi-component media may be written in form of

$$\sigma_{ka} = \partial W(\boldsymbol{E}) / \partial H_{ka}. \tag{3}$$

Then for first and second order approximation on deformation gradients there are the expressions

$$\sigma_{ij(1)} = \lambda_{ijkl} e_{kl(1)};$$

$$\sigma_{ij(2)} = \lambda_{ijkl} E_{kl(2)} + H_{im(1)} \lambda_{mjkl} e_{kl(1)} + v_{ijklmn}(e_{kl} e_{mn})_{(1)}. \tag{4}$$

Here

$$e_{ij} = \frac{1}{2}(H_{ij} + H_{ji}); \qquad H_{ij} = \nabla_j u_i = u_{i,j};$$

$$E_{ij(2)} = e_{ij(2)} + f_{ij(2)}; \qquad f_{ij(2)} = \frac{1}{2}(H_{mi} H_{mj})_{(1)};$$

$$\lambda_{ijkl} = \mu(\alpha\delta_{ij}\delta_{kl} + 2I_{ijkl}); \qquad I_{ijkl} + \frac{1}{2}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}); \qquad I_{ijklmn} = \frac{1}{2}(I_{ipkl}I_{jpmn} + I_{jpkl}I_{ipmn}),$$

$$\nu_2 = -\mu(\beta_2 + \beta_3); \qquad \alpha = \lambda / \mu, \tag{5}$$

For an ideally elastic material, the Piola-Kirchhoff stress at time t depends only on the deformation gradient matrix at time t, the functional Φ in (2) becomes a matrix-valued function of $\boldsymbol{H}(\boldsymbol{x},t)$. Correspondingly, the functional Φ becomes a matrix-valued function of $\boldsymbol{H}(\boldsymbol{x})$ only.

Since continuum considered here is suggested to be subjected to finite deformation Kirchhoff stress tensor and deformation gradients are used as field variables in a fixed reference state. Nonlinear problem abovementioned is investigated in second order approximation theory when the gradient deformation terms higher than second order are neglected. Five constant elastic potential in elastic problem and five time integral operator functionals in visco-elastic one are used to build overall constitutive relations for isotropic media. The well known inclusion problem in infinite visco-elastic matrix is a base of investigation. For micro-inhomogeneous media described by linear integro-differential equations with random kernels that are be reduced to linear integral equation by the use of a Green's function for appropriated problem for homogeneous comparison medium [5] in infinite domain.

As it's well known, the essential assumption in the Mori-Tanaka approach states [3.5], that the each inclusion behaves as isolated one in the infinite matrix and subjected to some effective stress field coinciding with the average stress in the matrix. Such an assumption allows uniquely define the effective elastic and visco-elastic properties of multi-component media. A nonlinearly visco-elastic composite medium is considered with stress free strains, which consists of a homogeneous matrix containing a homogeneous and statistically uniform random set of ellipsoidal inclusions having all the same form, orientation and mechanical properties. The main hypothesis of many micromechanical methods is used, according to which each inclusion is located inside a homogeneous stress and method of statistically conditional function and Mori-Tanaka approach can be applied. For a single inclusion such a micromechanical approach based on the Green function technique as well as on the interfacial Hill operators allows investigating the stress and strain concentration on the inter-phase surfaces.

More detailed considerations of the mechanical behavior of visco-elastic non-homogeneous media require the analysis of the interface between the different phases. These interfaces may represent for example weak interfacial layer due to imperfect bonding between the two phases. Therefore, to evaluate more accurately the visco-elastic effective properties of a composite, the behavior and structures of interfaces must be taken into consideration. At first the problem of a single uncoated and coated inclusion is considered inside an infinite matrix and then homogenization scheme proposed is implemented. As the result the overall constitutive equations are obtained.

$$\sigma(\boldsymbol{H}) = J^{-1}\boldsymbol{F}\boldsymbol{S}(\boldsymbol{H})\boldsymbol{F}^T; \qquad J = \det(\boldsymbol{F});$$

$$\boldsymbol{S}(\boldsymbol{H}) = \mu[\alpha I_1\boldsymbol{1} + 2\boldsymbol{E} + 3\beta_1 I_1^2\boldsymbol{1} + \beta_2 I_2\boldsymbol{1} + \beta_2 I_1(I_1\boldsymbol{1} - \boldsymbol{E}) + \beta_3(I_2\boldsymbol{1} - I_1\boldsymbol{E}) + \boldsymbol{E}^2];$$

$$\lambda = 3\sum_{r=1}^{n+1} c_r\kappa_r\kappa_{Ar} - 2\mu/3; \qquad \mu = 2\sum_{r=1}^{n+1} c_r\mu_r\mu_{Ar}; \qquad \nu_1 = \sum_{r=1}^{n+1} 3c_r\begin{bmatrix} l_A[2\lambda\kappa_A(3\kappa+\mu)_A + n_A f_A] + 9\nu_1\kappa_A^3 + \\ 6\nu_2 l_A\kappa_A(3\kappa+2\mu)_A + 8\nu_3 l_A^2 n_A \end{bmatrix}_r;$$

$$\nu_2 = \sum_{r=1}^{n+1} c_r\left[4\mu_A^2\left(\frac{1}{2}f_A + 3\nu_2\kappa_A + 4\nu_3 l_A\right) - \frac{1}{2}(3\lambda\kappa_A + 2\mu l_A)\right]_r; \; \nu_3 = \sum_{r=1}^{n+1} c_r\left[\frac{3}{2}\mu\mu_A(4\mu_A^2 - 1) + 8\nu_3\mu_A^3\right]_r. \tag{6}$$

The dependence of the effective properties from the visco-elastic properties of constituents is numerically investigated. The analysis is carried out by means of two approaches that primarily include investigating local fields of random stress and deformation under the external loading applied. Moreover, the influence of viscous properties of constituents is the subject of problem-solving.

References

[1] Christensen R.: Mechanics of composites. Wiley-Interscience, NY 1979.
[2] Hill R.: On constitutive macro-variables for heterogeneous solids at finite strain. *Proc. R. Soc. London* **A326**:131-147, 1972.
[3] Maslov B.P.: Stress concentration in non-compressible multi-component material. *Int. Appl. Mech.* **36,**N3:108-114, 2000.
[4] Rivlin R.S.: Material Symmetry Revisited *GAMM-Mitteilungen* **25**, Heft 1/2:109-126, 2002.
[5] Willis J.R.: Elasticity Theory of Composites *Mechanics of Solids. The Rodney Hill 60th Anniversary Volume* (eds. H.G. Hopkins, M.J. Sewell), pp.653-686, Pergamon Press, Oxford 1982.