MIGRATION AND INTERACTION OF TWO CONDUCTING PARTICLES FREELY IMMERSED IN A LIQUID METAL

Antoine Sellier
LadHyX, Ecole Polytechnique, 91128 Palaiseau Cédex, France

Summary We determine the rigid-body motions of two solid and conducting particles freely immersed in a liquid metal and subject to uniform ambient electric and magnetic fields. The advocated procedure resorts to a very few boundary-integral equations on the surface of the cluster and circomvents to calculate the electric field and the liquid flow in the unbounded fluid domain. The implemented numerical strategy will be presented together with the numerical results for several two-particle clusters.

MOTIVATIONS AND AIMS

Any solid and conducting particle freely suspended in a liquid metal of uniform conductivity $\sigma_l > 0$ and viscosity μ migrates when subject to uniform ambient electric and magnetic fi elds $\mathbf{E}$ and $\mathbf{B}$ (Kolin, 1953). Since this phenomenon may receive applications in impurities removal it is worth to determine the translational and angular velocities $\mathbf{U}$ and $\mathbf{\Omega}$ of a particle of prescribed but arbitrary shape and uniform conductivity $\sigma \geq 0$. Whilst a sphere of radius a admits the simple motion (Leenov & Kolin 1954)

$$\mathbf{U} = \frac{\sigma_l(\sigma - \sigma_l)}{3(\sigma + 2\sigma_l)} \frac{a^2}{\mu} (\mathbf{E} \wedge \mathbf{B}) \quad \text{and} \quad \mathbf{\Omega} = \mathbf{0} \tag{1}$$

non-spherical particles in general translate not necessarily parallel to $\mathbf{E} \wedge \mathbf{B}$ and rotate (Moffatt & Sellier 2002; Sellier 2003a, 2004). Because clusters may occur in practice it is also of interest to look at the challenging case of two close (and therefore interacting) particles $\mathcal{P}_1$ and $\mathcal{P}_2$ of uniform conductivity σ_1 and σ_2, respectively. In this direction the boundary formulation established for a single particle in Sellier (2004) has been recently extended to the case of two *insulating spheres* (Sellier 2003b). The present work shows how to cope with the general case of two *conducting and arbitrary-shaped* particles by solely solving a very few boundary-integral equations on the entire surface of the cluster.

GOVERNING EQUATIONS AND ADVOCATED STEPS

We consider, as sketched in fi gure 1, two solid and conducting particles $\mathcal{P}_1$ and $\mathcal{P}_2$ freely suspended in a Newtonian liquid metal of uniform viscosity μ and conductivity $\sigma_l > 0$.

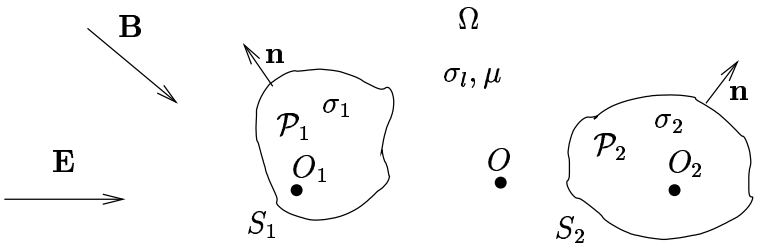

Figure 1. Two solid, conducting and arbitrary-shaped particles $\mathcal{P}_1$ and $\mathcal{P}_2$.

The particle $\mathcal{P}_n$ (n=1,2) has uniform conductivity $\sigma_n \geq 0$ and smooth boundary S_n. We denote by O_n a point attached to $\mathcal{P}_n$ and by $\mathbf{n}$ the unit outward normal on the entire surface $S = S_1 \cup S_2$. The two-particle cluster disturbs the ambient electric fi eld $\mathbf{E}$. More precisely, in $\mathcal{P}_n$ and in the liquid domain Ω the electric fi eld is $\mathbf{E} - \nabla\phi_n$ and $\mathbf{E} - \nabla\phi$, respectively. The unknown functions ϕ_n and ϕ obey the well-posed problem

$$\nabla^2\phi = 0 \ \text{ in } \ \Omega, \ \ \nabla^2\phi_n = 0 \ \text{ in } \ \mathcal{P}_n, \ \ \nabla\phi \to \mathbf{0} \ \text{ as } \ OM \to \infty \tag{2}$$

$$\sigma_l(\mathbf{E} - \nabla\phi).\mathbf{n} = \sigma_n(\mathbf{E} - \nabla\phi_n).\mathbf{n} \ \text{ and } \ \phi = \phi_n \ \text{ on } \ S_n \quad \text{for} \quad n = 1, 2. \tag{3}$$

Under the same physical assumptions of Moffatt & Sellier (2002) the liquid experiences a quasi-static creeping fbw described by its velocity and pressure fi elds $\mathbf{u}$ and p such that

$$\nabla.\mathbf{u} = 0, \ \ \mu\nabla^2\mathbf{u} = \nabla p - \sigma_l(\mathbf{E} - \nabla\phi) \wedge \mathbf{B} \ \text{ in } \ \Omega, \tag{4}$$

$$(\mathbf{u}, p) \to (\mathbf{0}, \sigma_l[\mathbf{E} \wedge \mathbf{B}].\mathbf{OM}) \ \text{ as } \ OM \to \infty, \ \ \mathbf{u} = \mathbf{U}^{(n)} + \mathbf{\Omega}^{(n)} \wedge \mathbf{O_nM} \ \text{ on } \ S_n \tag{5}$$

with $\mathbf{U}^{(n)}$ the velocity of O_n and $\mathbf{\Omega}^{(n)}$ the angular velocity of $\mathcal{P}_n$. Since our particles are freely immersed in the liquid we fi nally supplement (2)-(5) with the additional conditions (taking into account the Lorentz body force inside each particle)

$$\sigma_n \int_{\mathcal{P}_n} (\mathbf{E} - \nabla\phi_n) \wedge \mathbf{B} dv + \int_{S_n} \boldsymbol{\sigma}.\mathbf{n} dS = \mathbf{0} \quad \text{for} \quad n = 1, 2, \tag{6}$$

$$\sigma_n \int_{\mathcal{P}_n} \mathbf{O_n M} \wedge [(\mathbf{E} - \nabla\phi_n) \wedge \mathbf{B}] dv + \int_{S_n} \mathbf{O_n M} \wedge (\boldsymbol{\sigma}.\mathbf{n}) dS = \mathbf{0} \quad \text{for} \quad n = 1, 2 \tag{7}$$

with $\boldsymbol{\sigma}$ the usual stress tensor associated to the liquid flow $(\mathbf{u}, p)$.

Of course, for arbitrary-shaped particles $\mathcal{P}_1$ and $\mathcal{P}_2$ one needs to solve (2)-(7) by a numerical method. At the very first glance, one may also think about determining the unknown rigid-body motions $(\mathbf{U}^{(1)}, \boldsymbol{\Omega}^{(1)})$ and $(\mathbf{U}^{(2)}, \boldsymbol{\Omega}^{(2)})$ by appealing to the following steps (i)-(iii):

(i) Obtain ϕ_n in $\mathcal{P}_n$ and ϕ in Ω by solving (2)-(3).

(ii) Use the previously computed functions ϕ_n and ϕ to deduce the flow $(\mathbf{u}, p)$ solution to (4)-(5) for arbitrarily prescribed values of $(\mathbf{U}^{(1)}, \boldsymbol{\Omega}^{(1)})$ and $(\mathbf{U}^{(2)}, \boldsymbol{\Omega}^{(2)})$.

(iii) Finally, determine the rigid-body motions so that (6)-(7) hold through an iterative procedure appealing to the key step (ii).

Unfortunately, the fluid domain is unbounded and this prevents us from accurately achieving the key steps (i)-(ii) at a reasonable cpu time cost even by using a standard Finite Element solver.

In this work we therefore advocate and present a quite different approach that is free from the above drawbacks. The established procedure circumvents to calculate the functions ϕ_1, ϕ_2, ϕ and the liquid flow $(\mathbf{u}, p)$ in the unbounded fluid domain Ω. It rests on the use of adequately selected boundary-integral equations on the entire surface S of the cluster and permits us to compute the rigid-body motion of two arbitrary-shaped conducting particles at a reasonable cpu time cost. For conciseness, we only describe in this extended abstract the main steps of the work. These steps are:

1) We first establish a whole boundary formulation of the problem, i. e. show how it is indeed possible to obtain the rigid-body motion of each particle by solely solving boundary-integral equations on S.

2) For distant particles such that $\epsilon = (a_1 + a_2)/O_1O_2 \ll 1$ with a_n the length scale of $\mathcal{P}_n$ we asymptotically obtain each rigid-body motion in terms of the small parameter ϵ. This is achieved by appealing to the derived boundary formulation and permits us to quantify the very first correction to the motion of isolated particles.

3) For close particles the asymptotic solutions obtained in Step 2) are not valid any more and we therefore give and implement a suitable numerical strategy to numerically solve each encountered boundary-integral equation.

4) Finally, we present and discuss numerical results for a few two-sphere clusters of equal or inequal spheres for several settings $(\mathbf{E}, \mathbf{B})$ and $(\sigma_1/\sigma_l, \sigma_2/\sigma_l)$.

References

[1] KOLIN A.: An electromagnetokinetic phenomenon involving migration of neutral particles *Science*, **117**, 134–137, 1953.

[2] LEENOV D.,KOLIN A.: Theory of electromagnetophoresis. I. Magnetohydrodynamic forces experienced by spherical and symmetrically oriented cylindrical particles. *J.Chem.Phys.*, **22**, 683–688, 1954.

[3] MOFFATT H. K., SELLIER A.: Migration of an insulating particle under the action of uniform ambient electric and magnetic fields. Part 1. General theory. *J. Fluid Mech.***464**, 279–286, 2002.

[4] SELLIER A.: Migration of an insulating particle under the action of uniform ambient electric and magnetic fields. Part 2. Boundary formulation and ellipsoidal particles. *J. Fluid Mech.***464**, 335–353, 2003a.

[5] SELLIER A.: On particle-particle interactions in impurities removal. 4^{th} International Conference on Electromagnetic Processing of Materials. Lyon, France. 14-17 October 2003b.

[6] SELLIER A.: Migration of an insulating particle under the action of uniform ambient electric and magnetic fields. Part 3. Case of a conducting particle and numerical results. Submitted to *J. Fluid Mech.* 2004.