DYNAMIC STABILITY OF FUNCTIONALLY GRADED PLATE UNDER IN-PLANE COMPRESSION

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Summary Parametric vibrations of functionally graded plates subjected to in-plane time-dependent forces destabilizing the equilibrium state are analyzed. Nonlinear moderately large deflection equations taking into account a coupling of in-plane and transverse motions are used. Material properties are graded in the thickness direction of the plate according to volume fraction power law distribution. The asymptotic stability criteria are derived using Liapunov's direct method.

PROBLEM FORMULATION

Functionally graded materials have gained considerable attention in the high temperature applications. Functionally graded materials are composite materials, which are microscopically inhomogeneous, and the mechanical properties vary smoothly or continuously from one surface to the other. It is this continuous change that results in gradient properties in functionally graded materials (FGM). Many studies have examined FGM as thermal barriers. With the increased usage of these materials it is also important to understand the dynamics of FGM structures. A few studies have addressed this. Transient thermal stresses in a plate made of functionally gradient material were examined by Obata and Noda (1993).Vibration analysis of functionally graded cylindrical shells was performed by Loy, Lam and Reddy (1999). Recently, Lam, Liew, and Reddy (2001) presented dynamic stability analysis of functionally graded cylindrical shells under periodic axial loading. Consider the thin functionally graded rectangular plate with in-plane dimensions a and b, and thickness h. In-plane and transverse displacements are denoted by u, v, and w, respectively. Moderately large deflection equations taking into account a coupling of in-plane and transverse motions are used. Due to a small thickness rotary inertia terms are neglected. An oscillating temperature causes generation of in-plane time-dependent forces destabilizing plane state of the plate equilibrium. Material properties are graded in the thickness direction of the plate according to volume friction power law distribution. The viscous model of external damping with a constant coefficient ß is assumed. Taking into account the Kirchhoff hypothesis on non-deformable normal element and the Karman-type geometric nonlinearity the plate dynamics is described by partial differential equations system in the domain $O \bullet (0,a)\times(0,b)$. The membrane forces are stochastic with means equal to zero and known probability distributions. The processes are physically realizable and sufficiently smooth in order the solution of dynamics equations exists. We use the extensional, coupling and bending stiffnesses A_{ij}, B_{ij}, and D_{ij}, (i,j)=1,2,6) in the form

$$\left(A_{ij},B_{ij},D_{ij}\right)=\overset{h/2}{\underset{h/2}{\bullet}} Q_{ij}(z)\left(1,z,z^2\right)dz$$

where Q_{ij} are the reduced stiffnesses for plane isotropic materials. Relationships between in-plane forces (N_x, N_y, N_{xy}) and moments (M_x, M_y, M_{xy}) and the middle plane strains ($u_{,x}+1/2\, w_{,x}^2$, $v_{,y}+1/2\, w_{,y}^2$, $u_{,y}+v_{,x}+ w_{,x}\, w_{,y}$) and curvatures ($w_{,xx}$, $w_{,yy}$, $2w_{,xy}$) are described by the known constitutive equations. The effective elastic modulus, effective Poisson's ratio, and mass density ? of the functionally graded plate are denoted by E_{ef} , $?_{ef}$, and $?_{ef}$, respectively. In order to model the material properties of functionally graded materials, the properties must be both temperature and position dependent. This is achieved by using a rule of mixtures for the mechanical parameters E, ?, ?. The volume fraction is a spatial function and the properties of the constituents are functions of the temperature. The combination of these functions gives the effective material properties of functionally graded materials F_{ef} in terms of the appropriate properties of the ceramic and the metal, respectively, and V is the volume fraction of the ceramic constituent of the functionally graded material. A simple power law exponent of the volume fractions is used to describe the amount of ceramic and metal in the functionally graded material

$$V(z)=((2z+h)/2h)^q$$

where q is the power law exponent. The plate is assumed to be simply supported along each edge. The conditions imposed on displacements and internal forces and moments, called according to Almroth's (1966) classifications S2. The transverse motion of the plate is described by the nonlinear uniform equations with the trivial solution $w = w_{,t} = 0$ corresponding to the plane (undisturbed) state. In the paper a stochastic extension of Liapunov stability called the almost sure asymptotically stability of the trivial solution is analyzed. The crucial point of the method is a construction of a suitable Liapunov functional, which is positive for any motion of the analyzed system. The measure of distance between disturbed solutions and equilibrium state is chosen as the square root of Liapunov functional.

STABILITY ANALYSIS

The energy-like Liapunov functional has the form of a sum of modified kinetic energy and potential energy

$$V_n = T + \frac{1}{2}\int_O \left[-M_x w_{,xx} - M_y w_{,yy} - 2M_{xy} w_{,xy} + N_x \left(u_{,x} + 0.5w_{,x}^2\right) + N_y \left(v_{,y} + 0.5w_{,y}^2\right) + N_{xy}\left(u_{,y} + v_{,x} + w_{,x} w_{,y}\right) + \right.$$

$$\left. + \overline{N}_x w_{,xx} + \overline{N}_y w_{,yy} \right] dO \qquad T = \frac{1}{2}\int_O \left[w_{,t}^2 + 2\beta w w_{,t} + 2\beta^2 w^2 \right] dO$$

It may be observed that contrary to the linear or linearized case is the fourth order functional. It is assumed that the in-plane forces are periodic or stochastic non-white stationary and sufficiently smooth ergodic process. Therefore, it is legitimate to use the classical differentation rule. Upon differentiation with respect to time, substituting dynamic equations and using the boundary conditions we obtain the time derivative of Liapunov functional

$$\frac{dV_n}{dt} = -2\beta V_n + 2U_n$$

where the auxiliary functional U is given. Therefore, the stability analysis of the nonlinear system depends on the construction of the bound U_n = $?V_n$. The associate Euler equations are nonlinear in the case of the fourth-order functionals. It complicates a stability analysis and in order to obtain the analytical form of function we have to modify the variational problem. Therefore, our object is to find such second order functionals V* and U* that the inequality U^* = $?V^*$ will imply the inequality for the nonlinear problem. In order to do this we express functional in the form where V is the second order Liapunov functional for a linearized problem. Omitting in considerations the fourth order terms we obtain the lower estimation of V_n and the upper estimation of U_n. Solving the associated Euler equations for the modified linearized problem we find the function ?. If the in-plane processes are ergodic with the known probability distribution, the sufficient condition of the almost sure asymptotic stability is written as follows $\beta \leq E\lambda$, where E denotes the mathematical expectation. The stability region of the plate made of steel and zirconia for a zero-mean Gaussian force acting in x direction with the variance σ^2 is shown in Fig. 1.

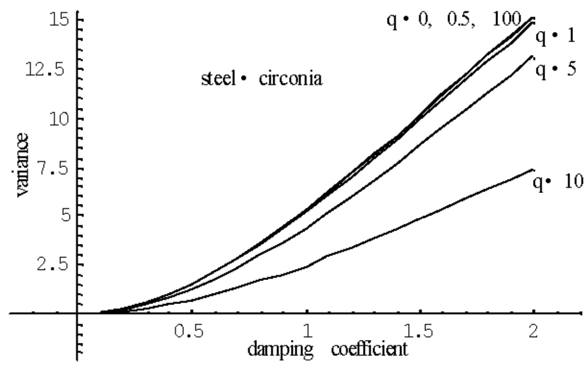

Fig. 1 Stability domains of quadratic plate unidirectionally loaded by in-plane time-dependent Gaussian force for the different power law exponents

CONCLUSIONS

The applicability of the direct Liapunov method has been extended to geometrically nonlinear functionally graded plates subjected to time-dependent, in-plane forces. The major conclusion is that the linearized problem should be modified to ensure the stability of nonlinear problem. The critical value of stability domains for intermediate values of power law exponent is substantially decreased.

References

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