THE STRESS ANALYSIS OF THE MULTILAYERED PLATES AND SHELLS WITH DEFECTS OF THE STRUCTURE

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Summary The laminated composite shells have high strength properties. However technological processes of their manufacture are difficultly automated, that causes the increased probability of manufacturing of thin-walled elements of structures with local defects. The local imperfections can produce essential influence on the strength of the total shell. This paper is devoted to such problems.

The theory of laminated anisotropic plates and shells is offered for the analysis of such situations The high order kinematics model reflects nonlinear character of distribution of displacements on thickness of rigid layers. These layers are connected by the thin glue layer It is supposed, that on some local area of the shell glue layer is absent, therefore in this area the unilateral contact between rigid layers is taken into account.
The contact problem of the mechanics of the non-uniform plates and shells with defects such as a local area of the non-glued layer are given in [1-3], where on the basis of the discrete approach the systems of the equations of the problem under condition of the non ideal contact of layers are formulated. In this work the theory multilayered anisotropic shell is offered on the base of the geometrically nonlinear theory of shells in a view of deformations of transverse shear.
The contact problem with aid of the mathematical theory [4] of variational inequalities is offered. The value of the Reissners energy functional of multilayered shell has view

$$R=\sum_{k=1}^{n}(\Pi_{(k)}+\mathrm{A}_{(k)})+\sum_{k=1}^{n-1}(\Pi_{[k]}+\mathrm{A}_{[k]}), \tag{1}$$

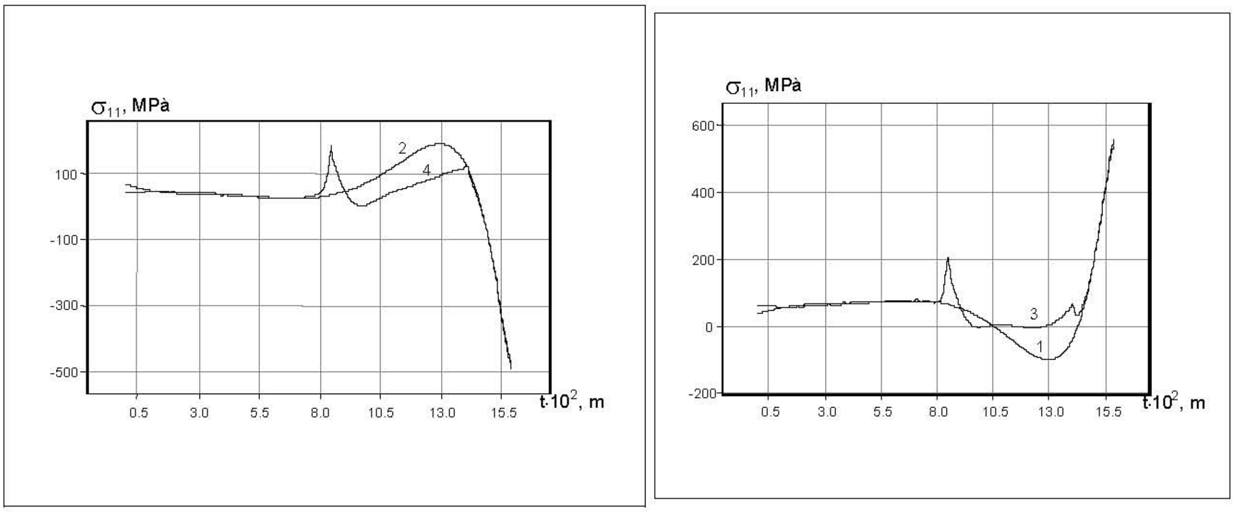
Where $\Pi_{(k)}$, $\Pi_{[k]}$ – potential energy of deformation k of a rigid layer and k the glue layer accordingly; $\mathrm{A}_{(k)}$, $\mathrm{A}_{[k]}$ – the work of external forces enclosed to k - rigid layer and k - soft layer of the shell.
The main variational inequalities on the base of functional energy (1) in which independent displacements and stresses are varied, allows [5] to receive system of the equations of the equilibrium of the multilayered shells, physical relations, boundary conditions. If between k and $k+1$ - layers of the shell the glue layer is absent, than contact pressure $\vec{q}_{(k)}$, $\vec{q}_{(k+1)}$. acts on the surface of interaction $S_z^{(k,k+1)}$ of these layers.
The contact pressure $\vec{q}_{(k)}=q^i_{(k)}\vec{r}_i^{(k)}+q^3_{(k)}\vec{m}^{(k)}$ is absent if the condition

$$(\vec{u}_z^{(k)}-\vec{u}_z^{(k+1)})\geq 0 \tag{2}$$

is satisfied in area of interaction of rigid layers.

As an example the stress analysis is considered for anisotropic toroidal shell loaded by internal pressure. The initial surface of shell is formed by rotation of a circle of radius r = 0.1m around an axis of rotation. The distance from the axis of rotation up to the circuit center R=0.4m. The shell is made from two of the reinforced layers by thickness h=5 mm and angles of the reinforcement on equator $\varphi^{(1)}=-60°,\varphi^{(2)}=60°$. It is considered, that on equator the conditions of symmetry are carried out, and the cross-section of the shell with coordinate $t_k=\pi r/2$ is rigidly jammed. The layers are

made from boron-epoxy tape of a composite material. The physical characteristics of boron filaments $E_v=4{,}2\cdot10^5$ MPa, $\nu_v = 0{,}21$, and epoxy binder $E_m=3{,}5\cdot10^3$ MPa, $\nu_m = 0{,}33$.
The stress analysis of the shell was calculated by the Godunov's method. The nonlinear boundary problem was reduced to the decision of a sequence of linear problems by the generalized Newton method.
The distribution of normal stresses σ_{11} in a direction of longitudinal coordinate t are shown on fig. 1à,b for internal pressure q=3 MPa. (3" - internal surface, "4" - outside surface).



a) Fig. 1. b)

On length of the shell at 0,085 m<t<0,14 m there is the non-glued area . Thus between rigid layers of the shell there is a unilateral contact. For comparison the distributions of normal stresses σ_{11} without defect area are shown ("1" - internal surface, "2" - outside surface). The analysis of curves 3, 4 proves presence of boundary effect in the non-glued area. The accepted mathematical model of anisotropic shell essentially defines results of numerical analysis.

References

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