SURFACE WAVES ON AN IMPERMEABLE BOUNDARY OF A POROELASTIC MEDIUM

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Summary The dispersion relation for surface waves on an impermeable boundary of a fully saturated poroelastic medium is investigated numerically in the whole range of frequencies. To this aim a linear simplified model of a two-component poroelastic medium is used. Similarly to the classical Biot's model, it is a continuum mechanical model but it is much simpler due to the lack of coupling of stresses. In the whole range of frequencies there exist two modes of surface waves corresponding to the classical Rayleigh and Stoneley waves. The numerical results for velocities and attenuations of these waves are shown for different values of the bulk permeability coefficient in different ranges of frequencies. In particular, we expose the low and high frequency limits, demonstrate the existence of the Stoneley wave in the whole range of frequencies as well as the leaky character of the Rayleigh wave.

PROCEDURE

The purpose of this work is to investigate the dispersion relation for surface waves on an impermeable boundary of a fully saturated poroelastic medium in the whole range of frequencies. Therefore we present the linear form of the model for a two-component poroelastic saturated medium. We show the governing equations and the construction of the solution for a semiinfinite medium with an impermeable boundary. One part is devoted to the presentation of the boundary conditions on such an interface between a porous medium and a vacuum. Afterwards we show the general dispersion relation and summarize results of an earlier work [4] for the high and low frequency approximations. The main part of the presentation concerns numerical aspects: first we indicate the applied numerical procedure and then we illustrate the numerical results for the normalized velocities and attenuations of the Rayleigh and Stoneley waves. It is known that surface modes of propagation in linear models result from the combination of bulk modes. Physically, this means that at any point of the boundary classical longitudinal and shear waves combine into the Rayleigh wave which must be slower than both bulk waves. The presence of the second longitudinal bulk wave P2 yields the existence of the second surface mode - the Stoneley wave which should be slower than the P2-wave - the slowest of bulk waves. Both quantities, velocities and attenuations, are shown for different values of the bulk permeability coefficient, π, in different ranges of frequencies. A decay of the Rayleigh wave velocity, mentioned in the book [1] has been confirmed in the range of small frequencies in spite of the lack of static coupling between components. Moreover we compare the behaviour of the two types of surface waves with the behaviour of two bulk waves: P1, and P2.

In this summary we only show a part of the presentation, namely the governing equations, the boundary conditions, and the numerical results for a chosen value of the permeability coefficient: $\pi = 10^7 \, \frac{\text{kg}}{\text{m}^3\text{s}}$.

MODEL

Within the linear model of a two-component poroelastic saturated medium the process is described by the macroscopic fields $\rho^F(\mathbf{x},t)$ - partial mass density of the fluid, $\mathbf{v}^F(\mathbf{x},t)$ - velocity of the fluid, $\mathbf{v}^S(\mathbf{x},t)$ - velocity of the skeleton, $\mathbf{e}^S(\mathbf{x},t)$ - symmetric tensor of small deformations of the skeleton and the porosity n. These fields satisfy the following set of linear equations

$$\frac{\partial \rho^F}{\partial t} + \rho_0^F \operatorname{div} \mathbf{v}^F = 0, \quad \left| \frac{\rho^F - \rho_0^F}{\rho_0^F} \right| << 1, \quad \rho_0^F \frac{\partial \mathbf{v}^F}{\partial t} + \kappa \operatorname{grad} \rho^F + \beta \operatorname{grad}(n - n_E) + \hat{\mathbf{p}} = 0, \quad \hat{\mathbf{p}} := \pi(\mathbf{v}^F - \mathbf{v}^S),$$
$$\rho_0^S \frac{\partial \mathbf{v}^S}{\partial t} - \operatorname{div}\left[\lambda^S (\operatorname{tr} \mathbf{e}^S)\, \mathbf{1} + 2\mu^S \mathbf{e}^S + \beta(n - n_E)\mathbf{1}\right] - \hat{\mathbf{p}} = 0, \quad \frac{\partial \mathbf{e}^S}{\partial t} = \operatorname{sym} \operatorname{grad} \mathbf{v}^S, \quad \left\| \mathbf{e}^S \right\| << 1, \quad n_E := n_0 (1 + \delta \operatorname{tr} \mathbf{e}^S), \qquad (1)$$
$$\frac{\partial (n - n_E)}{\partial t} + \Phi \operatorname{div}(\mathbf{v}^F - \mathbf{v}^S) + \frac{n - n_E}{\tau} = 0, \quad \left| \frac{n - n_0}{n_0} \right| << 1.$$

Here ρ_0^F, ρ_0^S, n_0 denote constant reference values of partial mass densities, and porosity, respectively, and κ, λ^S, μ^S, β, π, τ, δ, Φ are constant material parameters. The first one describes the macroscopic compressibility of the fluid component, the next two are macroscopic elastic constants of the skeleton, β is the coupling constant, π is the coefficient of bulk permeability, τ is the relaxation time and δ, Φ describe equilibrium and nonequilibrium changes of porosity, respectively. For the purpose of this work we assume β=0.

BOUNDARY CONDITIONS

In order to determine surface waves in a saturated poroelastic medium we need conditions for $z=0$. In the general case of a boundary between a saturated porous material and a fluid the boundary conditions were formulated by Deresiewicz & Skalak. We quote them here in a slightly modified form and for an impermeable boundary

$$T_{13}\big|_{z=0} \equiv T_{13}^S\big|_{z=0} = \mu^S\left(\frac{\partial u_1^S}{\partial z}+\frac{\partial u_3^S}{\partial x}\right)\Bigg|_{z=0} = 0, \qquad \frac{\partial}{\partial t}\left(u_3^F - u_3^S\right)\Big|_{z=0} = 0,$$

$$T_{33}\big|_{z=0} \equiv \left(T_{33}^S - p^F\right)\big|_{z=0} = c_{P1}^2\rho_0^S\left(\frac{\partial u_1^S}{\partial x}+\frac{\partial u_3^S}{\partial z}\right) - 2c_S^2\rho_0^S\frac{\partial u_1^S}{\partial x} - \kappa\left(\rho^F-\rho_0^F\right)\big|_{z=0} = 0,$$

where u_1^S, u_3^S are x-, and z-components of the displacement $\mathbf{u}^S$, respectively, and u_3^F the z-component of the displacement $\mathbf{u}^F$.

RESULTS FOR VELOCITIES AND ATTENUATIONS

Velocities	Attenuations

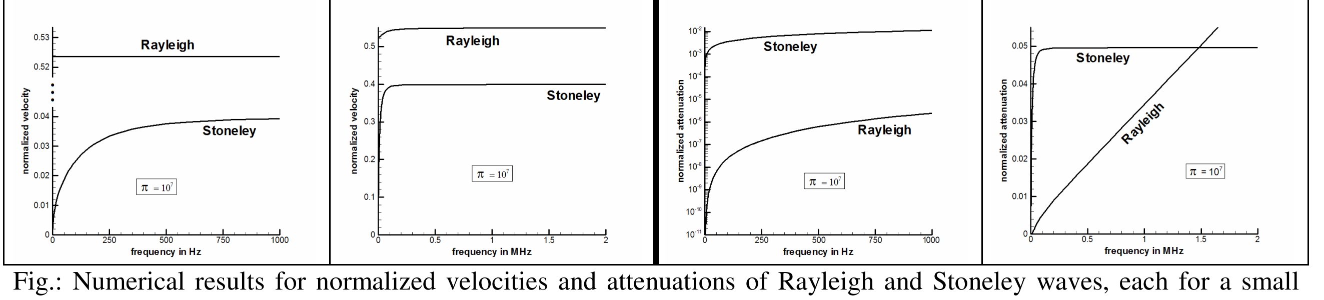

Fig.: Numerical results for normalized velocities and attenuations of Rayleigh and Stoneley waves, each for a small frequency range (left) and a large frequency range (right) for the permeability coefficient: $\pi = 10^7\ \frac{\mathrm{kg}}{\mathrm{m^3 s}}$.

CONCLUSIONS

Rayleigh

- the velocity of propagation of this wave lies in the interval determined by the limits ω→0 and ω→∞. The high frequency limit is app. 4.7% higher than the low frequency limit. The wave is always slower than the S-wave. As a function of ω it possesses an inflection point and it is slightly nonmonotonous,
- this nonmonotonicity appears in the range of small frequencies. The velocity possesses in this range a minimum whose size is very small. Interestingly, the minimum value remains constant for the different values of π. This means that the decay is not driven by the diffusion. Such a behaviour is also observed within Biot's model;
- the attenuation of this wave grows from zero for ω=0 to infinity as ω→∞. In the range of large frequencies it is linear (a constant positive quality factor). This means that it is a leaky wave.

Stoneley

- the velocity of this wave grows monotonically from the zero value for ω=0 to a finite limit which is slightly smaller than the velocity of the P2-wave. The growth of the velocity of this wave in the range of low frequencies is much steeper than this of Rayleigh waves similarly to the growth of the P2-velocity;
- both the velocity and attenuation of the Stoneley wave approach zero as √ω,
- the attenuation of the Stoneley wave grows monotonically to a finite limit for ω→∞ (zero quality factor). It is slightly smaller than the attenuation of P2-waves. Consequently, in contrast to the claims in the literature, the Stoneley wave is attenuated.

References

[1] Bourbie T., Coussy O., Zinszner B.: Acoustics of porous media, Editions Technip, Paris 1987.
[2] Deresiewicz H., Skalak R.: On uniqueness in dynamic poroelasticity. *Bull. Seismol. Soc. Am.* **53**:783-788, 1963.
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[4] Wilmanski K., Albers B.: Acoustic waves in porous solid-fluid mixtures, in: Dynamic Response of Granular and Porous Materials under Large and Catastrophic Deformations, K. Hutter, N. Kirchner (eds.), 285-314, Lecture Notes in Applied and Computational Mechanics, Vol.11, Springer, Berlin 2003.