BULK SOLITONS DO NOT DECAY IN ELASTIC WAVE GUIDES

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Summary Theoretical and experimental research has been performed to prove the existence of long bulk strain solitary waves produced by a laser-induced impact in nonlinearly elastic isotropic wave guides. In experiments with short wave guides (up to 30 length scale), we were not able to prove a lossless propagation of observed bulk solitons. *New experiments* on bulk soliton propagation in much longer wave guides (up to 150 length scale) allow to confirm that the solitons do not reveal any amplitude decay and shape transformation, while *any* linear or shock wave completely disappear at much shorter distance. It allows to propose a new NDT approach and a method to determine the 3d order elastic moduli of non-crystalline materials based on bulk elastic solitary waves propagation theory.

THEORY AND EXPERIMENTAL SETUP DESCRIPTION

Our aim is to prove the possibility of energy transfer by means of bulk solitary waves for very long distances without significant losses in 1D and 2D elastic wave guides. These wave guides have strong dispersion and made of materials with remarkable linear dissipation, that leads to disappearance any of linear or shock waves in short distances. Long nonlinear solitary strain waves (solitons) may be of considerable importance to study the time evolution of strains, the impact loading of materials with radiation and/or wear resistance, the nonlinear elastic parameters of materials etc. Nonlinear elastic features of these materials result in generation of strain solitons even under short-run and reversible (elastic) loading. Mathematical theory of long nonlinear waves in elastic wave guides is well developed in order to initiate the successful experiments in generation of strain solitons in solids. The nonlinear doubly dispersive equation (DDE in [1]) describes propagation of the long nonlinear wave of strain component $u(x,t)$ in a rod of radius R,

$$u_{tt} - c_0^2 u_{xx} = (1/2)\left[\beta\, u^2/\rho + \nu R^2 \left(c_0^2 u_{xx} - (1-\nu)u_{tt}\right)\right]_{xx}. \qquad (1)$$

where c_0 is the 'rod' sound velocity, $\beta = \beta(E,\nu;l,m,n)$ is a nonlinearity coefficient, (l,m,n) are the 3d order elastic moduli, ρ, ν are the specific density and the Poisson ratio, subscripts x, t denote derivatives in space and time, and u, β, ν may be of *any* sign. General travelling wave (TW) solution to (1), depending on $z = x \pm Vt$, is obtained in terms of the Weierstrass elliptic function $\wp$ as $u = u_0 + \wp(z + z_0; g_2; g_3)$, As appropriate limit it has also a particular solitary wave solution with velocity V and width Λ:

$$u = A\cosh^{-2}\frac{x \pm t\sqrt{c_0^2 + A\beta/(3\rho)}}{\Lambda}; \quad V^2 = c_0^2 + \frac{A\beta}{3\rho} > 0; \quad \Lambda^2 = 2(\nu R)^2\left[-\frac{1-\nu}{\nu} + \frac{3E}{A\beta}\right] > 0 \qquad (2)$$

In conventional solids with $0 < \nu < 1/2$ the soliton velocity lies in the interval: $c_0^2 < V^2 < c_0^2/(1-\nu) \equiv c_{\lim}^2$, and is limited from above, while no subsonic ($V < c_0$) solitary compression wave exists. Moreover, the value of V depends on nonlinearity of material and may be equal or *even greater* than the longitudinal sound wave velocity $c_l = \sqrt{(\lambda + 2\mu)/\rho}$ in the medium: $\nu \leq 1/3 \Rightarrow c_{\lim} \geq c_l$.

Two main applications were proposed for bulk solitons in mechanics: a new introscopy (NDT) method suitable for lengthy structures and a method to determine the 3d order elastic moduli of isotropic materials. Both are to be based on elastic energy transfer in a wave guide, and the main problem to be solved is for how long may a bulk solitary wave propagate without decay? In theory an ideal solitary wave is powerful and lossless, contrary to the 'common experience' with most of bulk longitudinal waves in mechanics of real solids.

We have performed numerous experiments on the generation and propagation of bulk strain solitary waves in polystyrene and plexiglas wave guides. The materials were selected due to unique combination of non-linear elasticity and optical properties allowing the translucent recording of wave patterns. The first generation of solitary nonlinear longitudinal strain wave (the soliton) was made in various 1D and 2D elastic wave guides by means of laser driven shock wave, the observation of waves was based on holographic interferometry. The experimental technique used for generation and observation of strain solitary waves in different wave guides was described, e.g., in [2], and is based on the laser generation and optical recording of the waves under study. The apparatus (see Fig.1) consists of a channel to produce the strain wave in a solid from a weak shock wave, which is induced by laser pulse evaporation of a metallic target placed nearby an entrance of the wave guide; a synchronizer; a holographic interferometer. This setup allows to record a wave pattern in and outside the transparent wave guide due to the wave induced density variations, which lead to the shifts of the carrier fringes on the resulting holographic interferogram. The relationship between the measured fringe shift ΔK on the interferogram inside the rod and soliton amplitude A was derived (see [1-2]):

$$A = -\lambda_1\, \Delta K \,/[2h[(n_1 - 1)(1 - 2\nu) + \nu(n_1 - n_0)]]$$

where λ_1 is the recording light wave length, n_1 , n_0 are refractive indices of solids and water, h is the rod thickness along the recording light path. We have demonstrated the successful generation and subsequent propagation of strain solitons in different rods and plates, observed the soliton amplification in a tapered wave guide, the soliton reflection from free or clamped ends of the rod, and soliton propagation in a wave guide embedded into another elastic medium. In numerical simulation the phase shift of the wave, its amplitude growth and the increase of a number of separate solitons were observed as distinctive characteristics suitable for introscopy of a wave guide.

RESULTS AND DISCUSSION

We certified the observed wave as a genuine strain solitary wave. It keeps the shape permanent, has no long wave of opposite sign behind, and its amplitude and width are proportional to the wave speed. In experiments with polystyrene and plexiglas we found that $\Lambda > 7R$.

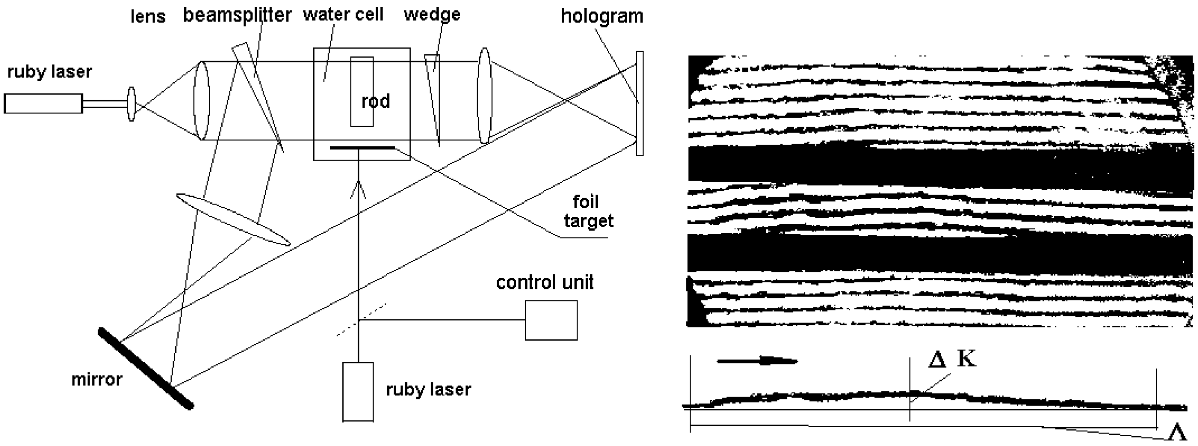

Figure 1. *Left*: Setup; *Right*: Interferogram of the soliton in the plexiglas rod. One of the fringes in the rod is extracted from the interferogram and shown below for convenience. Wave shape and amplitude both remain permanent at long distances.

Table. Parameters of solitons observed in polystyrene and plexiglas bars

Material	Amplitude, A	Width, Λ, mm	V, m/s	c_0,m/s	c_l,m/s
Polystyrene	$2.82 \cdot 10^{-4}$	32.5	≈ 2280	1870	2310
Plexiglas	$4.3 \cdot 10^{-4}$	32	≈ 2530	2060	2570

However, up to now we were limited by relatively short wave guides (up to 150 mm long, i.e., 30 scale units equal to R), that provoked some doubts in that the observed waves are genuine non-decaying solitary waves.

Now we demonstrate new experimental results on soliton behavior in lengthy polymeric wave guides (700 mm and longer). We prove that even *at very long distances* bulk solitons do not show any decrease of amplitude or shape transformation when propagating in an isotropic wave guide. On the contrary any of linear or shock waves in these materials disappear completely at much shorter distances. Numerical simulations were made also to demonstrate the advantages of the strain solitons usage for development of a new NDT method.

The measurements of (l, m, n) require to derive 3 independent algebraic relations, see [3]. In addition to (2) the coupled equations for the soliton propagation in the 2D wave guides (plates) and exact TW solutions to them were obtained in closed form in [1], providing an algebraic relation to the 3d order elastic moduli. The third independent relationship for nonlinear elasticity of solids was found as a solution to the solitary wave propagation problem in a wave guide made of the material under study embedded into another given elastic medium. The set of these three equations allows to calculate the values of the 3d order elastic moduli using our experimental data.

CONCLUSIONS

We proved for the first time in experiments an abnormally small decay rate (almost lossless propagation) of bulk longitudinal solitary deformation waves in solid wave guides in true consistency with the theory.

References

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