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MICROMECHANICS-BASED ELASTIC MODEL FOR FUNCTIONALLY GRADED MATERIALS WITH PARTICLE INTERACTIONS

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Summary A micromechanical framework is proposed to investigate effective elastic behavior of functionally graded materials (FGMs). Microstructurally, particles are randomly dispersed in the matrix with gradual transitions. The effect of pair-wise interactions between particles is taken into account for the local stress and strain fields by using the modified Green's function method. Homogenization of the local field renders relations between the averaged strain, strain gradient and external loading.

INTRODUCTION

Functionally graded materials (FGMs) are characterized for spatially varying microstructures created by non-uniform distributions of the reinforcement phase, as well as by interchanging the role of the reinforcement and matrix in a continuous manner [1,2]. They are typically manufactured by two phases of materials with different properties. Experimental observations show that the typical microstructure of FGMs contains a particle-matrix zone with discrete particles filled in continuous matrix, followed by a skeletal transition zone in which the particle and matrix phases cannot be well defined because the two phases are interpenetrated into each other as a connected network.
While FGMs have been designed and fabricated by diverse methods to achieve unique microstructures, very limited analytical investigations are available to tackle the spatial variation of microstructure. Conventional composite models such as the Mori-Tanaka method and the self-consistent method are directly applied to estimate the effective elastic responses of FGMs [1-4]. Because they were originally developed for homogeneous mixtures, those models are not able to capture the material gradient nature of FGMs. Furthermore, since no direct local interactions between particles are taken into consideration, they could not take into account the graded particle distribution for FGMs.
In this paper a micromechanical framework is proposed to investigate the effective elastic behavior of FGMs. Based on the Eshelby's equivalent inclusion method, the pair-wise particle interaction is collected for any two particles embedded in the matrix medium. Given a uniform loading on the upper and lower boundaries of FGMs, averaged strains in particles are derived by integrating pair-wise interaction contributions of all particles. In the course of derivation, the microscopic representative volume element (RVE) is constructed to reflect the microstructure of FGMs. A transition function is adopted in the skeletal transition zone. From the effective stress and strain fields distributed in the gradation direction of FGMs, the effective elasticity distribution is solved as a function of gradation direction.

MICROMECHANICAL ANALYSIS OF FGMS

Consider a typical FGM microstructure in Fig. 1 containing two phases A and B with isotropic elastic stiffness $\mathbf{C}^A$ and $\mathbf{C}^B$, respectively. The overall grading thickness of the FGM is t. In each graded layer, micro-particles are uniformly distributed with a two-dimensionally random setting so that the material layer is statistically homogeneous. While these micro-particles cannot be observed in the macroscopic scale, the volume fraction of phase A or B (for convenience, we use ϕ to denote the volume fraction of phase A) is gradually changed in the gradation direction X_3. The transition zone boundaries d_1 and d_2 are generally determined by the FGM fabrication process directly related to $\phi(X_3)$. To calculate the effective FGM elastic stiffness $\bar{\mathbf{C}}(X_3)$, a uniform far-field stress tensor $\boldsymbol{\sigma}^0$ is applied on the FGM X_3 boundary. Based on the equilibrium condition, the averaged stress should be equal to the far-field stress as

$$\boldsymbol{\sigma}^0 = \phi(X_3)\mathbf{C}^A : \langle\boldsymbol{\varepsilon}\rangle^A (X_3) + \left[1-\phi(X_3)\right]\mathbf{C}^B : \langle\boldsymbol{\varepsilon}\rangle^B (X_3) \tag{1}$$

For any material point $\mathbf{X}^0$ in the range of $0 \le X_3 \le d_1$, the corresponding microstructural RVE contains a number of micro-particles of the phase A embedded in a continuous matrix of the phase B so that the overall volume fraction of particle phase A and the its gradient should be consistent with the macroscopic counterparts $\phi\left(X_3^0\right)$ and $d\phi/dX_3\,|_{X_3=X_3^0}$.
The volume-averaged particle strains are collected based on the local strain fields in particles located at various graded layers. Specifically, particle's averaged strain in the center of RVE can be written in two parts: the elastic-mismatch interaction between the central particle and the matrix and the pair-wise interaction between the central particle and other particles. The first part can be solved by Eshelby's theory and the second part can be obtained by Mura's solution for two particles embedded in the infinite domain. Because all particles are statistically distributed in a random way, the probability of particle distribution can be introduced to statistically demonstrate the particle interaction effect. By applying Taylor series expansion of $\langle\boldsymbol{\varepsilon}\rangle^B(x_3)$ up to linear term in terms of x_3 and transferring the local coordinate of RVE to global coordinate of FGMs, we can integrate the particle interaction tensor and finally obtain particle's averaged

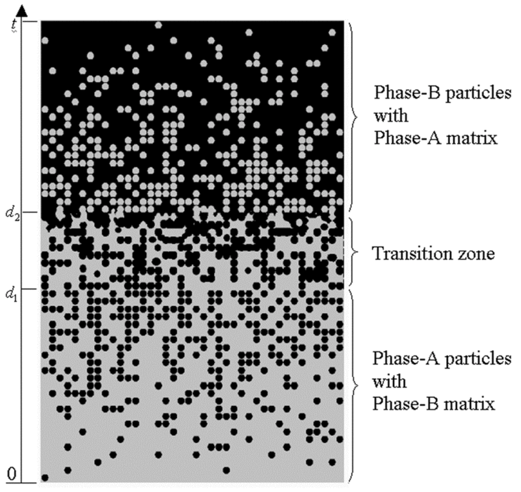

Fig. 1 Schematic illustration of a two-phase FGM sample

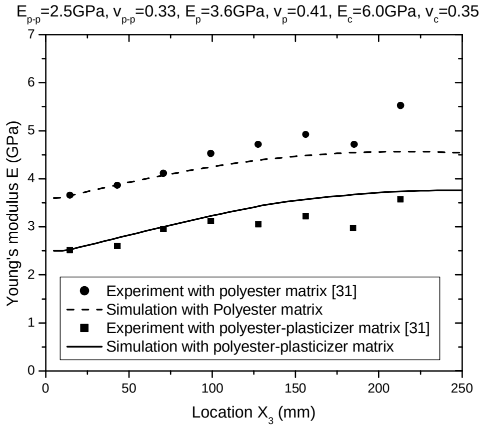

Fig. 2 Comparisons with experimental data [5]

strain along the gradation direction as:

$$\langle \boldsymbol{\varepsilon} \rangle^A (X_3) = (\mathbf{I} - \mathbf{P}_0 \cdot \Delta\mathbf{C})^{-1} : \langle \boldsymbol{\varepsilon} \rangle^B (X_3) + \phi(X_3)\Delta\mathbf{C}^{-1} \cdot \mathbf{D}(X_3) : \langle \boldsymbol{\varepsilon} \rangle^B (X_3) + \phi_{,3}(X_3)\Delta\mathbf{C}^{-1} \cdot \mathbf{F}(X_3) : \langle \boldsymbol{\varepsilon} \rangle^B_{,3} (X_3) \quad (2)$$

With the combination of Eqs. (1) and (2), the averaged particle strain tensor $\langle \boldsymbol{\varepsilon} \rangle^A (X_3)$ and the averaged matrix strain tensor $\langle \boldsymbol{\varepsilon} \rangle^B (X_3)$ in the FGM gradation direction X_3 can be solved in terms of the far-field stress $\boldsymbol{\sigma}^0$. Since Eq. (2) is an ordinary differential equation, we also need the appropriate boundary conditions. In the particle-matrix zone with $0 \le X_3 \le d_1$, the boundary at $X_3 = 0$ corresponds to the 100% matrix material, so the corresponding boundary conditions can be proposed as $\langle \boldsymbol{\varepsilon} \rangle^B (0) = \mathbf{C}^{B^{-1}} : \boldsymbol{\sigma}^0$. Thus the averaged strain tensors in both phases can be solved.

Similarly, in the other particle-matrix with the range of $d_2 \le X_3 \le t$, we can also calculate the averaged strain fields by the switch of matrix and particle phases. For the transition zone, a transition function is introduced so that the averaged strain of each phase (A or B) can be approximated as a cubic Hermite function appropriately contributed by the averaged strain of the same phase (A or B) from two particle-matrix zones. Thus the overall averaged strain tensor at each layer in the transition zone can be further obtained. From the relations of averaged stress and strain, we can easily derive Young's modulus and Poisson's ratio. It is noted that the proposed transition function satisfies the requirement that the effective FGM elastic fields and corresponding moduli should be bounded, continuous, and differentiable.

RESULTS AND CONCLUSIONS

From the above procedures, we can find that the proposed model considers direct interactions between particles and capture the material gradient nature of FGMs. In the particle-matrix zone, if the particle interaction terms are dropped, the proposed model is reduced to the Mori-Tanaka's model. To demonstrate the validity of the micromechanics-based particle interaction model, in Fig. 2 we compare the proposed model with the experimental data [5] for two types of FGM fabrications: cenospheres in the polyester matrix and cenospheres in the polyester-plasticizer matrix with volume fraction distributions $\phi = -0.4731 + 4.226\times10^{-3} X_3 - 8.666\times10^{-6} X_3^2$ and $\phi = -0.3729 + 4.561\times10^{-3} X_3 - 1.06\times10^{-5} X_3^2$, respectively. The thickness of the two FGMs is 250mm ($0 \le X_3 \le 250mm$). The phase Young's moduli and Poisson's ratios are given as: $E_p = 3.6GPa$, $v_p = 0.41$, $E_{p-p} = 2.5GPa$, $v_{p-p} = 0.33$ with the subscript p denoting the polyester matrix and $p-p$ representing the polyester-plasticizer matrix. Cenosphere particles are the hollow spheres made of aluminum silicates with the mean diameter of $127\mu m$ and wall thickness of $12.7\mu m$. During the simulation process, the hollow spheres are replaced by solid particles with the estimated Young's modulus and Poisson's ratio as $E_c = 6.0GPa$ and $v_c = 0.35$. With these parameters as input data, the effective Young's moduli of the FGMs are simulated and shown in Fig. 2 as a function of location. The proposed model compares well with the experimental results.

References

[1] Suresh S, Mortensen A.: Fundamentals of Functionally Graded Materials. IOM Communications Ltd, 1998.
[2] Miyamoto Y, et al.: Functionally Graded Materials: Design, Processing and Applications. Kluwer Academic Publishers, 1999.
[3] Vel SS, Batra RC.: AIAA J **40**:1421, 2002.
[4] Kim JH, Paulino GH.: Inter J Num Meth Eng: **58**:1457, 2003.
[5] Parameswaran V, Shukla A. J Mater Sci : **35**:21, 2000.