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LARGE EDDY SIMULATIONS OF DECAYING ROTATING TURBULENCE

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Summary Large eddy simulations of decaying, homogenous turbulence subjected to system rotation were performed using the Truncated Navier-Stokes method. It is observed that the nonlinear energy transfer from large to small scales is reduced by rotation, the energy decay is inhibited, the energy spectrum departs from the classical $k^{-5/3}$ form, and initially isotropic turbulence becomes anisotropic, with the anisotropy reflected in longitudinal integral length scales and directional stress tensors. We found that the latter effect is strongest at moderate rotation rates. At high Reynolds numbers and for sufficiently long times the Reynolds stress tensor and its invariants also become anisotropic, contrary to commonly reported results at low Reynolds numbers. The anisotropy may be responsible for the energy spectrum having k^{-3} rather than k^{-2} form found under the assumption of isotropy.

THE GOVERNING EQUATIONS AND THE NUMERICAL METHOD

The Navier-Stokes equations in a frame of reference rotating with an angular velocity Ω in the x_3 direction are:

$$\frac{\partial}{\partial t}u_i + \frac{\partial}{\partial x_j}u_i\, u_j = -\frac{1}{\rho}\frac{\partial}{\partial x_i}p + \nu\frac{\partial^2 u_i}{\partial x_j \partial x_j} - 2\Omega\varepsilon_{i3l}u_l, \tag{1}$$

where $\mathbf{u}$, p, ρ, and ν are the fluid velocity, pressure, density, and kinematic viscosity, respectively. Rotation considerably affects the dynamics and structure of these flows through the Coriolis force but classical turbulence models, developed for non-rotating flows, perform poorly in the presence of rotation. We have performed large eddy simulations of rotating turbulence using the truncated Navier-Stokes (TNS) approach, which is appropriate for rotating turbulence [1, 2]. TNS is effectively a sequence of low resolution direct numerical simulation (DNS) runs with a periodic processing of the solution that provides SGS dissipation, and between processing instants, the energy transfer from the physical scales is accomplished by the nonlinear interactions with the small, estimated scales. The method is easy to implement, does not use the concept of a SGS stress tensor and since only Navier-Stokes equations are solved, TNS satisfies all transformation properties of the Navier-Stokes equations.

For homogenous turbulence, we apply periodic boundary conditions in all three Cartesian directions and use a Fourier representation for all dependent variables. The N-S equation (1) transformed to spectral space is solved numerically using a well-known pseudo-spectral method with an explicit second-order predictor-corrector time stepping scheme. To alleviate time step restrictions viscous and Coriolis terms are integrated analytically using the integrating factor technique [3].

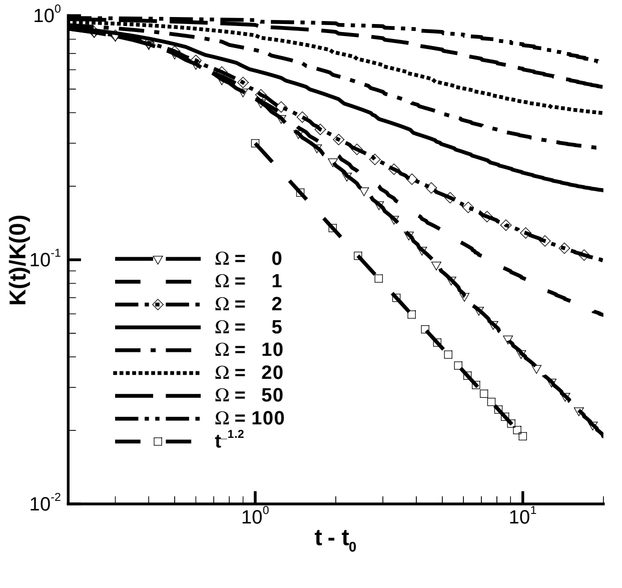

Figure 1. TNS: History of energy decay on a log-log scale for nominally infinite Re. Long dashed line with gradients: $\Omega = 0$; dashed-dotted-dotted line with diamonds: $\Omega = 1$; dashed line: $\Omega = 2$; solid line: $\Omega = 5$; dashed-dotted line: $\Omega = 10$; dotted line: $\Omega = 20$; long dashed line: $\Omega = 50$; dashed-dotted-dotted line: $\Omega = 100$; dashed line with squares: $t^{-1.2}$ law.

RESULTS

The TNS model was used to perform simulations at two Reynolds numbers for rotating turbulence using the same initial condition of Horiuti [4]. For low Reynolds number, TNS provides results in agreement with theoretical analysis, DNS and other LES data for cases with and without rotation. We confirmed that the energy decay is inhibited by rotation. The $k^{-5/3}$ form appeared for the non-rotating case but a spectrum close to k^{-2} was observed for moderate rotation rate $\Omega = 10$. At high rotation rates, the energy spectrum obeyed a linear viscous decay law. The coupling effect between nonlinear interaction and rotation triggers the anisotropy which is reflected in longitudinal integral length scales and a directional stress tensor. The presence of anisotropy is conveniently quantified using macro- and micro-Rossby numbers Ro^L and Ro^ω, respectively, defined as follows: $Ro^L = u'/(2\Omega L)$, where L is the integral length scale and u' is the rms turbulent velocity; $Ro^\omega = \omega'/(2\Omega)$, where ω' is the rms vorticity. We found that anisotropy effects are strongest at moderate rotation rates for Rossby numbers approximately in the range between $Ro^L < 1$ and $Ro^\omega > 0.1$.

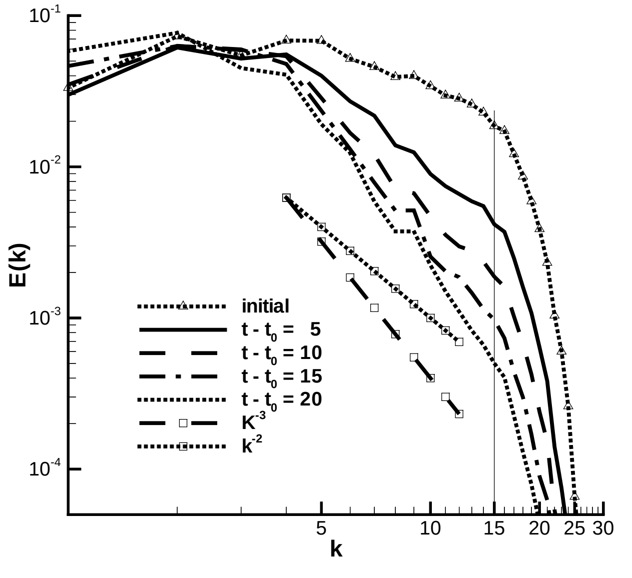

Figure 2. TNS: Energy spectrum at different time for $Re = 4 \times 10^{15}$, $\Omega = 20$. Dotted line with deltas: initial condition; solid line: $t - t_0 = 5$; dashed line: $t - t_0 = 10$; dashed-dotted line: $t - t_0 = 15$; dotted line: $t - t_0 = 20$; dashed line with squares: k^{-3} spectrum; dotted line with squares: k^{-2} spectrum.

At high Reynolds numbers, the energy decay in a case without rotation is consistent with $t^{-1.2}$ decay law but, in general, the rotation effects are stronger than at low Reynolds numbers . The energy decay is inhibited more for all rotation rates considered (Fig. 1). Even for very rapid rotation $\Omega = 100$, the evidence of nonlinear interactions modified by rotation was obvious and its behavior differed from the simple linear viscous decay observed at lower Reynolds number. For sufficiently long calculation time, we found that the Reynolds stress tensors and their invariants also became anisotropic, which is different from the results obtained in the short time, low Reynolds number runs. At high Reynolds numbers, the energy spectra close to k^{-2} and k^{-3} are observed, depending on the level of anisotropy (Fig. 2). At earlier times, when anisotropy is weak, the spectral slope is -2, consistent with phenomenological theories of rotating turbulence [5]. At later times, anisotropy is strong and the spectral slope tends toward value of -3. The latter result is consistent with analysis of Cambon [6] who showed that a k^{-3} spectrum can be derived in the asymptotic limit of quasi-infinite Reynolds number and quasi-zero Rossby number by using a wave-turbulence statistical model derived from a generalized EDQNM theory.

References

[1] J. A. Domaradzki and K. Horiuti, "Similarity modeling on an expanded mesh applied to rotating turbulence", Phys. Fluids **13**, 3510 (2001).

[2] J. A. Domaradzki, K. C. Loh and P. P. Yee, "Large Eddy Simulation Using the Subgrid-Scale Estimation Model and Truncated Navier-Stokes Dynamics", Theoret. Comput. Fluid Dynamics **16**, 1-30 (2002).

[3] P. K. Yeung and Y. Zhou, "Numerical study of rotating turbulence with external forcing", Phys. Fluids **10**, 2895 (1998).

[4] K. Horiuti, "Rotation tansformation and geometrical correlation of SGS models", in *Modern Simulations Strategies for Turbulent Flw*, edited by B. J. Geurts (R. T. Edwards, Philadelphia, 2001).

[5] Y. Zhou, "A phenomenological treatment of rotating turbulence", Phys. Fluids **7**, 2092 (1995).

[6] C. Cambon, personal communication, 2003.