

*21<sup>st</sup> International Congress of Theoretical and Applied Mechanics 2004***Non-Reflecting Boundary Condition for Direct Aeroacoustic Computation**X. M. Li, R. C. K. Leung and R. M. C. SoDepartment of Mechanical Engineering, The Hong Kong Polytechnic University  
Hung Hom, Kowloon, Hong Kong**Short Summary**

A new non-reflecting boundary condition (NSPML), based on perfectly matched layer concept applied to compressible Navier-Stokes equations, is proposed for direct aeroacoustic computation. The boundary condition is designed to absorb acoustic waves incident at all angles. The NSPML is validated with aeroacoustic flows of increasing complexity, and found that it provides higher non-reflectivity than existing boundary conditions but at a much lower computational cost.

**Extended Summary**

Non-reflecting boundary condition is an important issue in computational aeroacoustic studies. In direct aeroacoustic computations, high-order schemes are required to resolve the small amplitudes of the acoustic waves in order to distinguish them from the large amplitude fluid dynamic fluctuations due to the unsteady flows. High-order schemes provide low numerical dissipation to resolve the acoustic wave propagation, but require precise boundary conditions to avoid spurious numerical errors generated at the computational boundaries. A numerical non-reflecting condition at all the computational boundaries is necessary; otherwise the spurious erroneous waves reflecting from the boundaries would contaminate the solution in the computational domain, decrease the computational accuracy, and may even drive the solutions towards a wrong time-stationary state.

Because of its simplicity, the classical one-dimensional non-reflecting characteristics-based boundary condition (NRCBC) is very popular [1]. However, when the physical waves are two-dimensional and incident at any angle towards the boundaries, only the normal components are adequately absorbed and significant numerical error waves are reflected. Recently, two approaches of numerical boundary conditions, namely perfectly matched layer (PML) method [2] and the Freund's buffer zone technique [3], have been proposed in the search for truly non-reflecting boundary conditions for two-dimensional aeroacoustics problems. Hu [2] derived a stable PML formulation for the linearized Euler equations in unsplit physical variables, which essentially forces the amplitudes of two-dimensional acoustic waves to zero in a PML region beyond the physical domain. Freund [3] rewrote the Navier-Stokes equations with additional damping terms, which act to suppress all the flow unsteadiness and force the flow towards a prescribed uniform flow in a buffer zone beyond the physical domain.

In this paper, a PML-based boundary condition for Navier-Stokes equations (NSPML) is proposed and its performance in aeroacoustic computation is assessed. Benchmark aeroacoustic flows of increasing complexity are computed to validate the NSPML. From the computation of a Gaussian pulse propagating radially outwards in a square domain, the two-dimensional non-reflectivity of the NSPML has been proven excellent (Figure 1). In general, propagating vortices play a critical role in sound generation, absorption and diffraction in unsteady flows. Therefore, the boundary conditions should, while providing non-reflectivity for the acoustic propagation, enable the vortices passing through them without any error waves generated at the boundaries. This capability has been tested by solving a canonical problem of convection vortex interacting with a Gaussian pulse on a uniform grid. The non-reflectivity of NSPML and the associated computational cost are compared with Freund's technique, NSCBC and a reference solution, which is computed with the domain size double. The NSPML zone is 10-grid wide. Figure 2 shows the time history of the error at the outlet boundary. Obviously the NSPML gives the least error, less than NSCBC and Freund's technique by two and one orders of magnitude respectively. In fact the performance of the NSPML is optimal in computational cost. The same NSPML accuracy can be obtained with a 40-grid wide Freund's buffer zone but the computational time has to be doubled. Direct aeroacoustic computations of a laminar boundary layer flow over an open cavity are carried out (Figure 3). It is found that the NSPML allows boundary layers passing through the computational boundaries without creating significant error waves. Furthermore, for the same number of grid points, the accuracy of NSPML is much better than Freund's buffer zone technique.

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**References**

- [1] Poinot T. J., Lele S.K: Boundary Conditions for Direct Simulations of Compressible Viscous Flows. *J. Comp. Phys.* **104**: 104-129, 1992.

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- [2] Hu F. Q.: A Stable, Perfectly Matched Layer for Linearized Euler Equations in Unsplit Physical Variables. *J. Comp. Phys.* **173**: 455-480, 2001.
- [3] Freund J. B.: Proposed Inflow/Outflow Boundary Condition for Direct Computation of Aerodynamic Sound, *AIAA J.* **35**: 740-742, 1997.

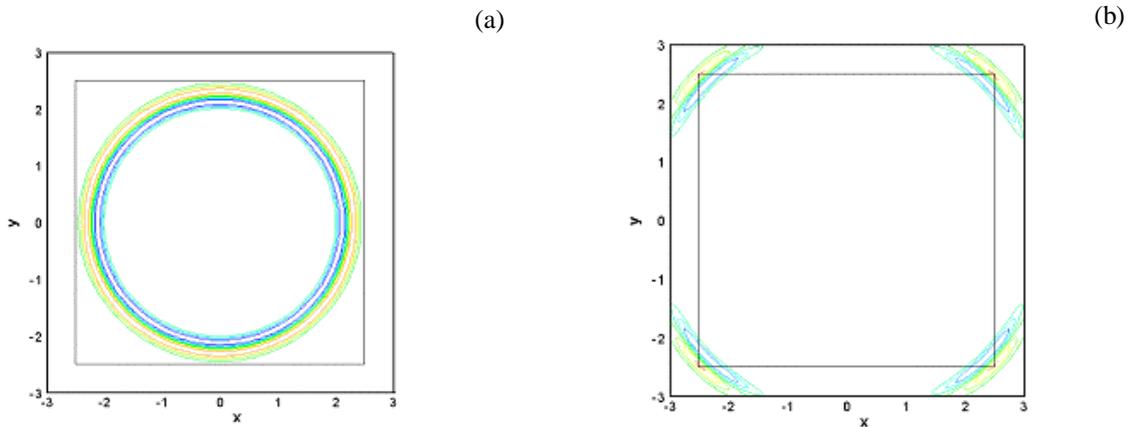


Figure 1. Computation of propagating Gaussian pulse with NSPML. (a)  $T' = 0.12$ ; (b)  $T' = 0.18$

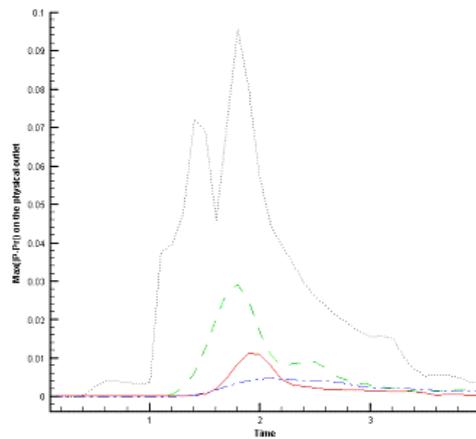


Figure 2. Time history of the maximum error in pressure fluctuations at outlet. —, NSPML (10 grids); --- Freund's buffer zone (10 grids); - · - · -, Freund's buffer zone (40 grids); ·····, NSCBC.

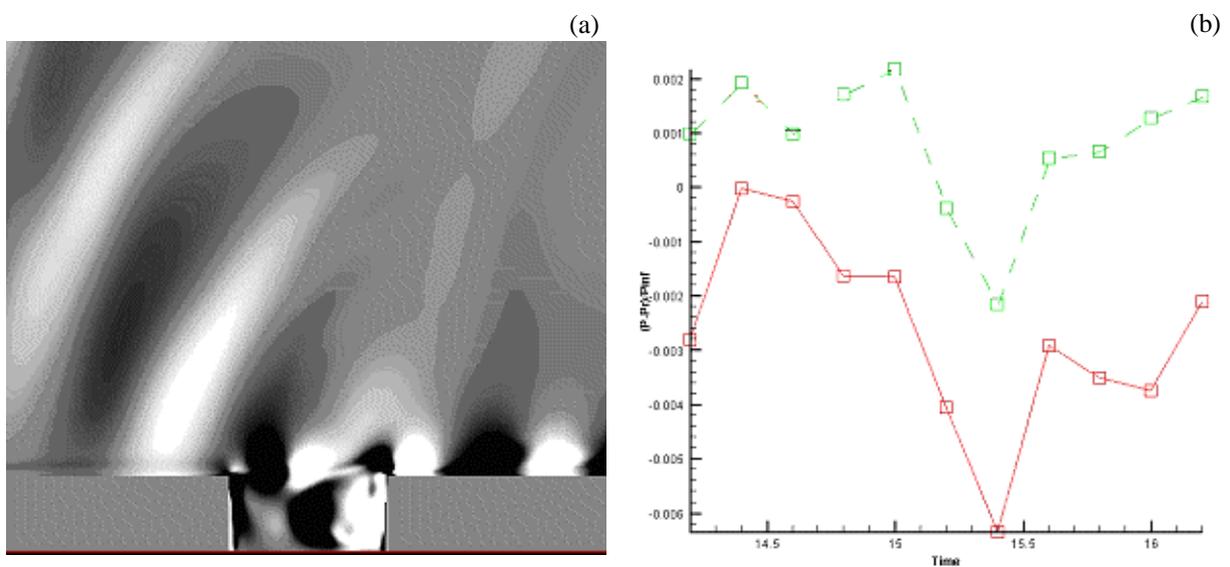


Figure 3. Computation of open cavity flow with NSPML. (a) Pressure wave; (b) time history of pressure error at the mid-point of domain outlet; —□—□—, NSPML; - □ - □ -, Freund's buffer zone.