Extended summary of the proposed ICTAM2004 lecture V. Shtern 7 October 2003

BIFURCATION OF CONICAL MAGNETIC FIELDS

Vladimir Shtern

University of Houston, TX 77204-4006, USA

1. Introduction. According to "anti-dynamo" theorems [1, 2], the axisymmetric dynamo is impossible. This result seems paradoxical because observed magnetic fields are nearly axisymmetric as for stars and planets [3] so for cosmic jets erupting from active galaxy nuclei [4]. The paradox was bypassed in two ways. Braginsky showed that even a slight asymmetry is sufficient for the dynamo to occur [2]. Another bypass is that a flow being unsteady and three-dimensional at small scales can generate a nearly steady and axisymmetric large-scale magnetic field [3].

In contrast to these bypasses, the present study deals with exactly axisymmetric steady states and shows that a magnetic field appears in a magnetic-free conical flow via a pitchfork bifurcation. This seemingly contradicts to the anti-dynamo" theorems, however, there is no contradiction indeed as conditions of the theorems do not hold for conically similarity flows.

Conical similarity is a feature of a wide family of solutions of the Navier-Stokes equations. This family includes the Schlichting, Landau, and Squire swirl-free round jets, the Long swirling, thermal convection near a point source of heat and gravity [5], and many other flows.

Streamlines and magnetic lines are open in a conical flow: they go to and from infinity or the singularity point. This feature is crucial as Cowling [1] emphasized that his anti-dynamo theorem is not applicable for a flow with open lines. Also, the proof by Braginsky [2] based on the condition that the magnetic induction, **H**, decays at infinity as r^{-3} or faster is not applicable to conical flows where both **H** and **v** decay weaker being proportional to r^{-1}; r is the distance from the flow origin.

Thus, on one hand, the theorems do not rule out a possibility of dynamo occurrence in conical flows. On the other hand, an MHD bifurcation in a conical flow can be not necessarily interpreted as dynamo.

2. Reduction to ODDE. An advantage of the conical similarity is the reduction of governing equation to ordinary differential equations (ODE) that radically eases the analysis.

The conical similarity means that

$$\{v_r, v_\theta, v_\phi\} = \nu r^{-1}\{u, u/\sin\theta, \Gamma/\sin\theta\},\ T = T_\infty + \gamma r^{-1}\vartheta,$$

$$p = p_\infty + \rho_\infty \nu^2 r^{-2} q,\ \{H_r, H_\theta, H_\phi\} = h\nu r^{-1}\{R, \Theta/\sin\theta, \Phi/\sin\theta\},\quad (1)$$

where p is pressure, T is temperature, $\{v_r, v_\theta, v_\phi\}$ and $\{H_r, H_\theta, H_\phi\}$ are the velocity and magnetic field components in spherical coordinates $\{r, \theta, \phi\}$ (Fig. 1), ρ is density, ν is viscosity, h is a scale constant, and γ characterizes the heat flux from the origin. Dimensionless functions u, u, Γ, ϑ, q, R, Θ, and Φ depend only on $x = \cos\theta$.

Suppose that a point source of gravity is located at the coordinate origin and neglect self-gravitation of the ambient fluid. This yields that the acceleration due to gravity is $\mathbf{g} = -\mathbf{e_r}\,\delta/r^2$. Here $\mathbf{e}_r$ indicates the outward radial direction, and δ characterize the strength of **g**. We apply the Boussinesq approximation, $\rho/\rho_\infty = 1 - \beta(T - T_\infty)$, β is the coefficient of thermal expansion.

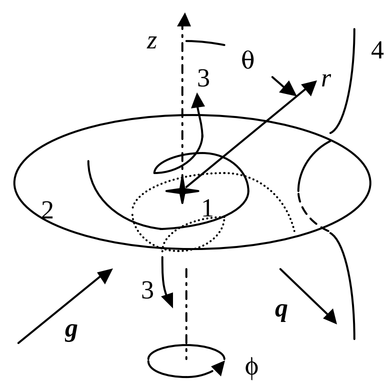

Figure 1. Schematic of a flow near a point source 1 of gravity ***g*** and heat flux ***q*** with accretion plane 2. Typical stream 3 and magnetic 4 lines are shown.

Substituting (1) reduces the MHD equations [1] to:

$$(1-x^2)\psi' + 2x\psi - \tfrac{1}{2}\psi^2 = F - \Theta^2/2, \quad (2a)$$
$$(1-x^2)F''' = Ra\psi\vartheta + 2\Gamma\Gamma' - 2\Phi\Phi', \quad (2b)$$
$$(1-x^2)\Gamma'' = \psi\Gamma' - \Theta\Phi', \quad (2c)$$
$$(1-x^2)\vartheta' = Pr\psi\vartheta, \quad (2d)$$
$$(1-x^2)\Theta'' = Bt(\psi\Theta' - \psi'\Theta), \quad (2e)$$
$$(1-x^2)\Phi'' = Bt[\psi\Phi' - \Theta\Gamma' + 2(\psi'\Phi - \Theta'\Gamma) + 2x(\psi\Phi - \Theta\Gamma)/(1-x^2)], \quad (2f)$$

where $\psi = -u$ ($u = -\psi'$, $R = \Theta'$); $Ra = \beta\gamma\delta/(\kappa\nu)$, $Pr = \nu/\kappa$, and $Bt = \nu/\nu_m$ are the Rayleigh, Prandtl, and Batchelor numbers, respectively; κ and ν_m are the thermal and magnetic diffusivities.

3. MHD bifurcation in the bipolar swirling jet. At Ra=0, consider a *vortex*-sink motion prescribed on the accretion plane (2 in Fig. 1), i.e., the boundary conditions at $x = 0$ are: $u = Re_p$, $u = 0$, and $\Gamma = Re_s$, where Re_p and the swirl Reynolds number, Re_s, characterize the accretion and vortex strengths. The velocity on the axis is bounded if $\psi = F = F' = \Gamma = 0$ at $x = 1$.

The magnetic field must satisfy the conditions, $\Phi(0)=\Theta'(0)=0$ (symmetry) and $\Phi(1)=\Theta(1)=0$ (regularity).

This problem has a solution with $\Theta=\Phi\equiv0$, that describes a swirling flow that converges (for $Re_p < 0$) to the axis near the disk and goes away from the origin along the axis (curve 3 in Fig.1). As the accretion ($-Re_p$) increases, a magnetic field appears via the pitchfork bifurcation, as Fig. 3 shows. The abscissa, $Al^{1/2}=0$, is the axis of plot symmetry and solutions with $Al^{1/2}>0$ and $Al^{1/2}<0$ have just opposite direction of the magnetic field; Al is the magnetic-to-kinetic energy ratio on the disk

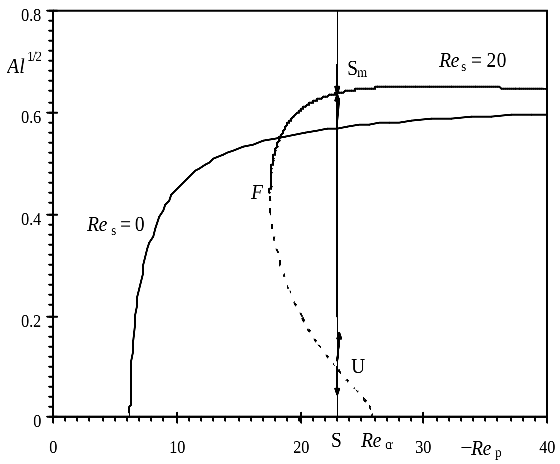

Figure 2. Bifurcation of magnetic field in a vortex-accretion flow.

We see that the MHD bifurcation is supercritical in a swirl-free (Re_s=0) flow and becomes subcritical when the swirl is sufficiently strong (e.g. at Re_s=20). Arrows show the time-evolution direction of a disturbed magnetic field. The presence of fold point *F* means that as the accretion increases and decreases, transformations between magnetic-free and MHD states occur via hysteretic transition in a flow with a strong swirl.

4. MHD bifurcation in the buoyancy flow. Now consider a bipolar outflow that develops via onset of thermal convection [5]. It appears that not only the convection, but also a magnetic field bifurcates in a converging-to-the-axis flow as Fig. 3 shows at Bt = 0.18. Line E corresponds to the equilibrium state of rest. At point *T* (Ra = 24), the trans-critical bifurcation of convection occur. No MHD bifurcation occurs in the diverging flow (lower inset and curve *La* at $\psi_{min}<0$). The converging flow (upper inset) is unstable (broken curve *FT*). As ψ_{max} reaches 4 (at fold point *F*), the jet velocity becomes unbounded and the fluid-sink singularity develops on the axis. In this singularity flow (curve *Tu*), the MHD (supercritical pitchfork) bifurcation occur at point *B*. In the MHD flow (curve *MT*), the magnetic field grows, consumes the kinetic energy, and suppresses the sink singularity at point *L* as *Ra* increases.

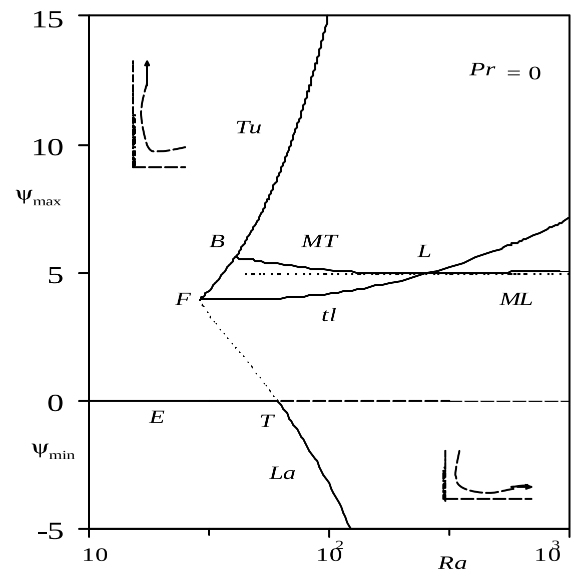

Figure 3. Bifurcations in a buoyancy flow.

Curve *tl* is a boundary between regular (below) and singular (above) flow states. Curve *ML* corresponds to a regular MHD flow. It is interesting that this solution branch is remote from other regular flow states.

5. Conclusions. This study has shown that self-sustained magnetic fields can develop via bifurcation in jet-like flows. A necessary condition for the MHD bifurcation is the presence of accretion in a flow. In the swirl-free accretion flow, the self-induced magnetic field eventually (as $-Re_p$ increases) suppresses the bipolar jet, but the jet can remain strong in the vortex-accretion flow (§3). In the buoyancy-induced flow (§4), the jet remains strong even in s swirl-free MHD state. Thus, our results reveal that the bifurcation of magnetic field is typical of jet-like flows and, therefore, can contribute to the development of long-range magnetic fields in cosmic jets as well.

[1] Cowling, T. G. 1957 *Magnetohydrodynamics*. New York, Interscience.
[2] Braginsky, S. I. 1964 Self-excitation of magnetic field during the motion of a highly conducting fluid. *Sov. Phys. JETP* **48**, 1084-1098.
[3] Moffat, H. K. 1978 *Magnetic field generation in electrically conducting fluids*. Cambridge University Press.
[4] Ferrari A. 1998 Modeling extragalactic jets. *Annu. Rev. Astron. Astrophys.* 36, 539-598.
[5] Shtern, V. & Hussain, F. 2001 Generation of collimated jets by a point source of heat and gravity. *J. Fluid Mech.* **449**, 39-59.

<< session << start