# ELASTIC-PLASTIC LARGE DEFORMATION ANALYSIS OF 2D FRAME STRUCTURE

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*Summary* This paper presents a numerical simulation method, so called vector form intrinsic finite element (VFIFE) method that can conduct the analysis of 2D frame structure under elastic-plastic large deformation. This new method can simultaneously calculate large rigid body motions and large geometrical changes of a structural system consisting of multiple continuous bodies. The essence of the method includes a set of equations of motion for the modeling nodes of the system, an explicit time integration scheme, the use of a deformation coordinate system to dissect rigid body and deformation displacements, and the use of a convected material reference frame to handle the deformation. Numerical examples of frame structure having large elastic-plastic deformation states under static and dynamic excitations are demonstrated to verify the accuracy and effectiveness of this newly proposed method.

## INTRODUCTION

The development of the Finite Element Method (FEM) is originated from the matrix analysis and slope deflection method of the structural mechanics. It promoted the concept of member-connection of structure to the continuous medium. The equilibrium condition is satisfied within all the members (elements) according to the deformation modes characterized by the connections (nodes) and a set of linear equations is obtained by equilibrating internal forces assembled from surrounding members (elements) of each connection (node). This intrinsic concept of physically modeling of the structural system obtains good results in analyzing continuous medium by using constant triangle elements. However, this member-connection modeling of linear structure admitted difficulties when it was applied to the higher order finite element, plate/ shell and 3D deformable bodies. The finite element method entered its new era after adopting the theory of the calculus of variation and some numerical techniques of approximation and discretization. The FEM currently known to the researchers can be identified as a mathematical modeling of continuous medium and is a computational modeling of Ritz-Galerkin analysis. Nevertheless, the analysis method based on the theory of the calculus of variation and the balance of energy inside the simply connected continuous medium still has its limitations on solving some real world problems especially with very large rigid body motion and multiple discontinuities.

## A VECTOR FORM INTRINSIC FINITE ELEMENT METHOD

A new computational method so called the Vector Form Intrinsic Finite Element (VFIFE) is developed by Ting etc. [1- 4] to handle engineering problems with the following characters: (1) containing multiple deformable bodies and mutual interactions, (2) material non-linearity and discontinuity, (3) large deformation and arbitrary rigid body motions of deformable body. Since the conventional FEM based on variational method requires the total virtual work to be zero but dose not require the balance of forces at nodes. These unbalanced residual forces will do some non-zero work under virtual rigid body motion and cause the inaccuracy and un-convergence of the calculation results. The computation procedure and some concepts of this VFIFE method are similar to the FEM. But the major difference is that the VFIFE does not apply the variational principle on the stress expressed equilibrium equations in its formulation. Instead, VFIFE maintains the intrinsic nature of the finite element method and makes strong form of equilibrium at nodes, the connections of members. All the forces balanced at each node are obtained from the principle of virtual work and the associated deformations are satisfied with the compatibility condition of deformation. In this method (see Fig. 1), explicit formulation is used to avoid the difficulties that are caused by iterations of material non-linearity and an incremental theory based on the material reference frame of current configuration is used to define the stress-strain relations. In the calculation process, the external loads are applied to the deformable body incrementally. These incremental forces apply to a continuous body with stress and motion. All the material properties, stress distribution, particle velocities and geometry are defined from the calculation results of previous time step. The motion path and loading history of material points in the system are independent to the incremental type computational algorithm. The history dependency is inherited in the adjusting of parameters of material model during the calculation. Hence, we can call this explicit calculation is based on the convected material frame. The incremental theory may not be so complete and accurate as the total deformation theory since errors are introduced in every incremental step. But, we are benefited from the simplification and convenience it provides to the code development.

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To verify the capabilities of VFIFE method, some problems with large deformation were studied. Consider a cantilever modeled by frame elements, and is subjected to constant end moment. The beam should coil with constant curvature.

Figure 2 shows simulations done by the VFIFE method exactly match the analytical solutions. The elastic-plastic behavior of a free ring subjected to impulse loading on its upper surface is simulated. Figure 3 shows the prediction matches the experimental data very well. Figure 4 shows the shapes and trajectories of fragments after fracture due to impulse loading. It is clear that the VFIFE method can simulate the problems with very large deformation better than the conventional methods.

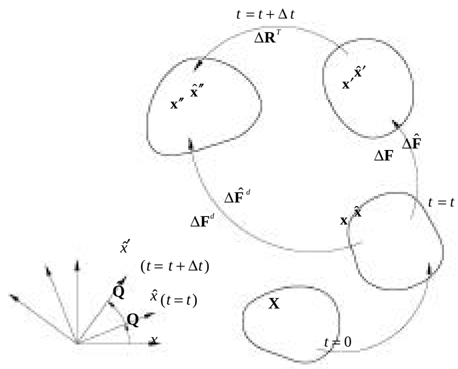


Figure 1.

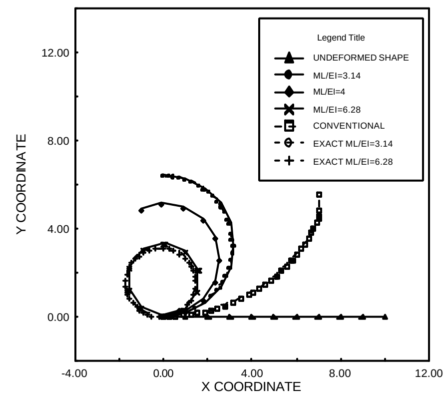


Figure 2.

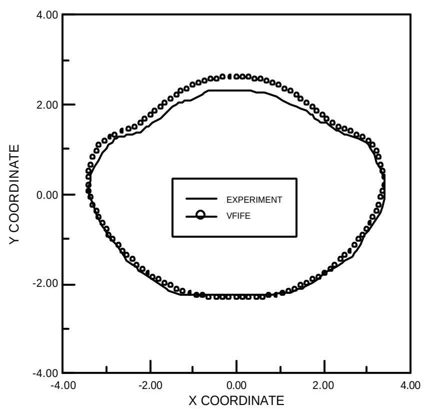


Figure 3.

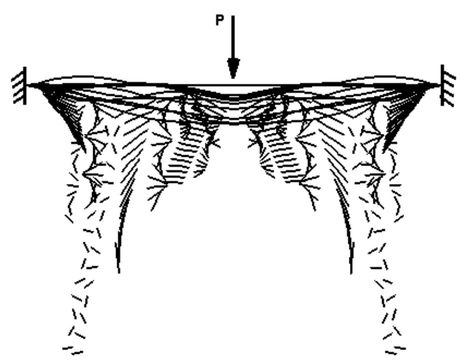


Figure 4.

## CONCLUSIONS

In this paper, a convected material frame is used to define strain, stress and virtual work. Based on this frame system, a new incremental analysis procedure is developed for the study of motions that involve very large deformation, displacements and fragmentation behaviors that are difficult to be conducted by the conventional FEM. This explicit procedure is named as vector form intrinsic finite element (VFIFE) method. The stability and accuracy of this method were verified by examples. Since the calculation of the VFIFE method is carried on each discrete node, it is very easy to couple this method with other numerical schemes to study mechanics problems containing solid- fluid- temperature-electromagnetic interactions. Besides, the explicit formulation of the VFIFE method allows it to model the discontinuous and nonlinear dynamic behaviors of material easily.

### References

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