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MULTISCALE ANALYSIS OF SCATTERED ELASTIC WAVES BASED ON THE LIPPMANN–SCHWINGER EQUATION

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Summary A method for the multiscale analysis of scattered elastic waves by means of the Lippmann–Schwinger equation is developed. A multiscale decomposition is defined herein in order to examine how the effects of fluctuations of the wave field are conveyed to the far field by Green's function. The numerical results were found to clarify the scale effects of the wave field on scattering of waves.

INTRODUCTION

Although a great deal of research concerning scattering of elastic waves is in the literature; however, the way in which small-scale fluctuation of the wave field influences the scattering of waves remains unclear. It is convenient to express the solution of the wave field in terms of the Lippmann–Schwinger equation (Colton and Kress, 1998) in cases in which the medium has a small-scale fluctuation. In the present paper, the Lippmann–Schwinger equation together with a multiscale decomposition (for example, Hughes, 1995) is applied in order to examine the scale effects of a medium on scattering of elastic waves.

THEORETICAL FORMULATION

Consider an infinite wave field in which the properties of the field vary continuously. For simplicity, the presence of only the S wave is assumed and the shear modulus in the medium is as follows:

$$\mu(x) = \mu_0 + \mu_1(x), \quad x \in \mathbb{R}^n, \quad \text{supp } \mu_1(x) \subset B \tag{1}$$

where $\mu(x)$ is the shear modulus, μ_0 is a constant and $\mu_1(x)$ is the fluctuation of the shear modulus around μ_0 and B is a compact set in $\mathbb{R}^n$. Let the fluctuation be made up of two parts. Namely,

$$\mu_1(x) = \mu_l(x) + \mu_s(x), \quad \text{supp } \mu_s \subset B_s \subset B \tag{2}$$

where μ_l is the shear modulus showing a large-scale variation, while μ_s is that for the localized small scale variation.

The first step of the formulation is to express the solution of the wave field in the form of the integral equation (the Lippmann–Schwinger equation) as follows:

$$u(x) = f(x) - \int_B a(x,y)u(y)dy, \quad x, y \in \mathbb{R}^n \tag{3}$$

where u is the displacement field, f is the free-field response of the wave field and $a(x,y)$ is the kernel of the integral equation which is constituted by $\mu(y)$ and Green's function for the homogeneous wave field. In the following, Eq. (3) is simply expressed as $u = f - Au$ for convenience.

Next step is the application of the multiscale decomposition to the solution of Eq. (3) such that

$$u(y) = u_l(y) + u_s(y), \quad y \in B, \quad B_s \subset \text{supp } u_s \subset B \tag{4}$$

where u_l is the large-scale solution and u_s is the small-scale solution which covers the area of the small-scale fluctuation. This type of decomposition becomes possible by means of the scaling function and wavelets of compact support. For example, u_l and u_s can be expressed as follows:

$$u_l(y) = \sum_{k\in\mathbb{Z}} \alpha_k \phi_{m,k}(y), \quad u_s(y) = \sum_{m'\geq m} \sum_{k\in\mathbb{Z}} \beta_k \psi_{m',k}(y) \tag{5}$$

where $\phi_{m,k}$ and $\psi_{m',k}$ are the Haar scaling function and wavelet (Williams and Amaratunga, 1994). According to the decomposition, the projections P and Q can be defined such that $u_l = Pu$, $u_s = Qu$ and $P \perp Q$. The decomposition of the displacement field leads to the following two integral equations.

$$u_s + QAu_s = Qf - QAu_l \tag{6}$$

$$u_l + PAu_l = Pf - PAu_s \tag{7}$$

According to Eq. (6), an operator M such that $u_s = M(f, u_l)$ can be defined if $\|QA\| < 1$. Then, Eq. (7) becomes

$$u_l + PAu_l = Pf - PA\; M(f, u_l) \tag{8}$$

from which the large-scale solution is derived.

NUMERICAL EXAMPLES

Figure 1 shows the analyzed model for 1-D wave propagation problem, in which a thin soft layer is embedded in the medium and a plane incident wave is propagating toward the soft layer. Properties of the material for the medium are described in Fig. 1 and the spatial variations of the shear modulus are shown in Fig. 2. The frequency of the analysis is 1 Hz. There are two cases for numerical examples. A small-scale fluctuation of the rigidity of the medium is imposed on Case-2. Figure 3 shows the comparison of the displacement between the two cases. For both cases, the convergence of the solutions using the Haar scaling function and wavelet is preliminarily examined. It was found from the examination that the size of the support of u_s covering the fluctuated area did not affect the total field solution, if the solution converged. The order of the resolution of Eq. (5) for Case-2 was found to be $m' = 7$ for the satisfactory level of accuracy. A difference in the amplitude in the backward side and that of phase lag in the forward side can be seen in Fig. 3. Figure 4 shows the comparison of the components of the scattering of the wave due to the large and small-scale solutions. According to the comparison, the amplitudes of both scale solutions are comparable outside the soft layer, although the amplitude of the small-scale solution is very small inside the layer. The small-scale solution is found to have a significant role outside the soft layer as the large-scale solution has.

CONCLUSIONS

Multiscale decomposition of the wave field was carried out for the Lippmann–Schwinger equation to examine the scale effects of fluctuation of the medium on scattering of waves. The usage of the scaling function and wavelet basis ensured the multiscale additive decomposition here. The numerical results showed that the small-scale solution also had as significant a role for scattering of waves as the large-scale solution.

References

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[3] Williams, J.R. and Amaratunga, K.: *Int. J. Num. Method. in Engineering*, **37**, pp. 2365-2388, 1994.

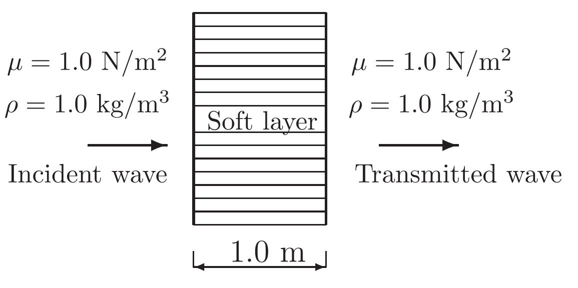

Figure 1. Analyzed model.

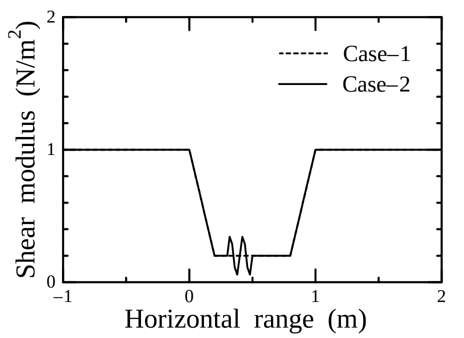

Figure 2. Spatial variation of the shear modulus.

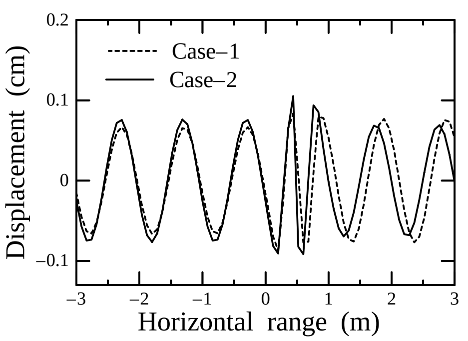

Figure 3. Comparison of displacement between the two cases.

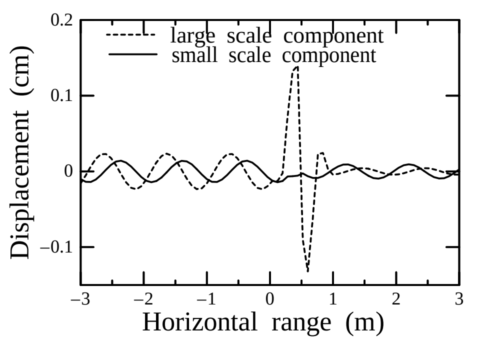

Figure 4. Comparison of the two scale solutions.