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# DISSIPATIVE EFFECTS ON PROPAGATION OF THE ACOUSTIC SOLITARY WAVES

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*Summary* This paper examines dissipative effects on the acoustic solitary waves propagating in an air-filled tube with an array of Helmholtz resonators. The dissipation is brought about by wall friction through boundary layers and by jet loss in resonators' throat. For pressure profiles measured experimentally, numerical simulations are carried out to identify the respective effects. Discussions are included on how the dissipative effects are reduced to realize a pulse as closely as possible to the solitary wave in lossless limit.

## INTRODUCTION

The acoustic solitary wave can be propagated in an air-filled tube if a periodic array of Helmholtz resonators is connected to the tube axially [1-4]. The solitary wave is a localized pulse consisting of compression phase only and propagating steadily at a subsonic speed [2]. It is made possible without forming a shock by the action of dispersion that the array yields. Experiments have been performed to verify the existence of the solitary waves [3,4].

In reality, however, dissipative effects are unavoidable more or less so that the solitary waves in lossless limit would be masked. The dissipation is brought about by friction through boundary layers developing on the tube wall and by jet loss when air flows through narrow throat of the resonator. Effects due to the diffusivity of sound are negligible because the acoustic Reynolds number is extremely high. For pressure profiles measured experimentally, numerical simulations are carried out by solving an initial-value problem to evolution equations derived previously [1] to identify the respective effects. To realize a pulse close to the solitary wave, discussions are given on how the dissipative effects are reduced.

## EXPERIMENTAL PRESSURE PROFILES

Experiments have been carried out by using an air-filled tube of inner diameter $2R$ $(= 80$ mm) and of length $l_t$ $(= 10.6$ m). The resonators of volume $V$ $(= 49.8\ \mathrm{cm}^3)$ with throat of inner diameter $2r$ $(= 7.11$ mm) and of length $L$ $(= 35.6$ mm) are connected to the tube in array with axial spacing $d$ $(= 50$ mm). The total number of the resonators connected is 212. The natural angular frequency of the resonator $\omega_0$ is given by $\sqrt{a_0^2 B/L_e V}$ where $a_0$, $B$ and $L_e$ denote, respectively, the sound speed, the cross-sectional area of the throat and the effective length of the throat $L + 2 \times 0.82r$ with end corrections. It takes 242 Hz at temperature $24.8\ {}^\circ$C where the experiments have been made.

An initial pulse is generated by driving impulsively a plane piston mounted at one end of the tube. The other end of the tube is closed by a flat plate. The form of the initial pulse is far away from the solitary wave. Figure 1 shows the temporal profile of the initial pulse measured at a position distant about 0.4 m from the piston where $p_0$ and $p'$ denote, respectively, the equilibrium (atmospheric) pressure (= $0.1010$ MPa) and the excess pressure over $p_0$. The pulse is propagated down the tube and reflected by the flat plat to return to the piston. Then reflected by the piston, it continues to propagate back and forth in the tube, decaying out eventually by dissipation. In this process, it is observed that the initial pulse evolves into a pulse similar in form to the solitary wave.

Figure 2 shows the profile measured at a position distant about 5.2 m from the piston after reflected three times by the flat plate and twice by the piston. The thick line and thin line indicate, respectively, the profiles measured and given by the solitary-wave-solution with the peaks coincided with each other. It is seen that the profile except for a hump and a tail on the right-hand side of the peak agrees with the solution very well. This agreement motivates us to identify whether another solitary wave might emerge from the hump by the dispersion or dissipative effects would be accumulated.

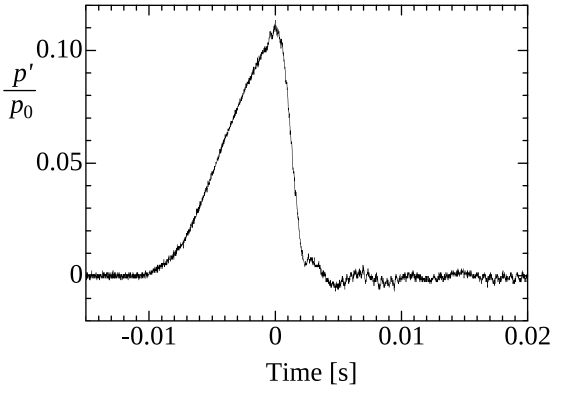


**Figure 1.** *Temporal pressure profile of the initial pulse measured at a position distant about 0.4 m from the piston.*

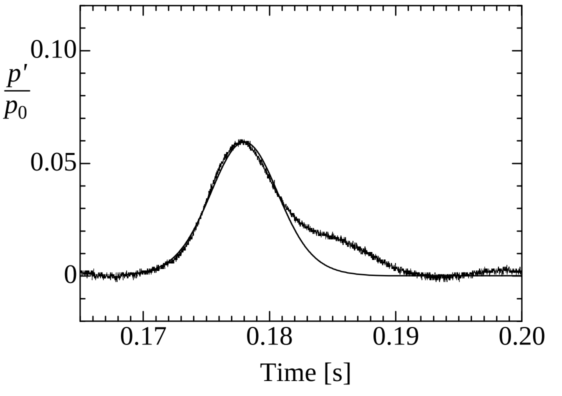


**Figure 2.** *Temporal pressure profiles at a position distant about 5.2 m from the piston where the thick and thin lines indicate the profile measured and the one of the solitary wave, respectively.*

## NUMERICAL SIMULATIONS

In order to answer this, numerical simulations are performed by solving the following evolution equations:

$$\frac{\partial f}{\partial X} - f\frac{\partial f}{\partial \theta} = -\delta_R \frac{\partial^{\frac{1}{2}} f}{\partial \theta^{\frac{1}{2}}} - \frac{\partial g}{\partial \theta}, \tag{1}$$

and

$$\frac{\partial^2 g}{\partial \theta^2} + \delta_r \frac{\partial^{\frac{3}{2}} g}{\partial \theta^{\frac{3}{2}}} + g = f + \tfrac{1}{2}\kappa N, \tag{2}$$

where $f$ and $g$ denote, respectively, the excess pressure in the tube and in the cavity, while $X$ and $\theta$ denote, respectively, the axial coordinate and the retarded time in a frame moving with the sound speed, all quantities being appropriately normalized; the hereditary effects due to the boundary layers on the tube wall and throat wall are expressed in terms of the fractional derivatives of 1/2- and 3/2-order, respectively, $\delta_R$ and $\delta_r$ being positive constants much smaller than unity; the jet loss is represented by the second term in $N$ given by

$$N = \left(\frac{\gamma - 1}{\gamma + 1}\right)\frac{\partial^2 g^2}{\partial \theta^2} - \frac{2V}{(\gamma+1)BL_e}\left|\frac{\partial g}{\partial \theta}\right|\frac{\partial g}{\partial \theta}, \tag{3}$$

$\gamma$ being the ratio of specific heats and $\kappa$ the size parameter of the array defined as $V/Ad\ (\ll 1)$ with $A = \pi R^2$.

An initial-value problem is solved by imposing an initial value of $f$ at $X = 0$, which is taken by fitting the initial profile in figure 1 with an appropriate function. The axial coordinate $X$ measures distance along which the pulse has travelled in the tube. Although (1) and (2) are derived originally for uni-directional propagation in a tube of infinite length, it is revealed that they still can be used to interpret the real propagation of the pulse in the bounded tube. Only difference lies in slight phase shifts where the profiles simulated are lagged behind the ones measured. The phase shifts result from reflection at both ends since it may be regarded as head-on collision of two identical solitary waves in a tube of infinite length.

The simulations are carried out for three cases. Firstly the lossless case is solved, for reference, by ignoring both the boundary-layer effects and the jet loss. Of course, the solitary wave is the steady-progressive-wave solutions in this case. Secondly only the boundary-layer effects are taken into account, and finally the full equations are solved. In the lossless case, it is found that the initial pulse evolves into a pulse like the solitary wave and no hump appears. When the boundary-layer effects come into play, the peak pressure becomes smaller and a tail tends to develop. But no hump is seen. When the jet loss comes in, the peak pressure is further reduced and a hump emerges. Thus the hump observed is identified to be brought about by the jet loss.

## DISCUSSIONS

Now that the evolution equations can simulate the real evolution quantitatively well, it implies that the dissipative effects can be controlled to achieve a pulse as closely as possible to the solitary wave. To reduce the boundary-layer effects, it is preferable to use a wider tube and throat. In addition, the kinematic viscosity should be reduced by raising equilibrium pressure $p_0$. On the other hand, the jet loss appears in (2) through the parameter $\kappa V/(\gamma+1)BL_e$, which is determined by geometry of the resonator only. Therefore the value of $\kappa$ should be reduced by taking longer axial spacing, while the resonators with smaller value of $V/BL_e$ should be used. In the present experiment, $\kappa$ takes 0.198. But $\kappa V/(\gamma+1)BL_e$ takes a large value 2.50 in spite of that the term of jet loss is regarded as being small in (2).

In view of these, new simulations are made by assuming the throat diameter and its length to be double in size, and the value of $\kappa$ to be half. Then the parameter of the jet loss takes 0.304. Furthermore $p_0$ is set to be 10 MPa. Taking the same initial pulse as the one in figure 1, it is found that the initial pulse evolves into a single pulse much closer to the solitary wave. Since the boundary-layer effects are now suppressed, the tail in this case may be brought about by the dispersion and a new solitary wave might emerge from it if a further long-time behaviour would be pursued.

## CONCLUSIONS

Making the numerical simulations, the respective dissipative effects due to the boundary layers and the jet loss can now be identified. Although they decay the pulse in common, the former gives rise to the tail, while the latter yields the hump. But their effects are limited to trailing behaviour of the pulse and the leading behaviour is well described locally by the solitary waves in lossless limit.

### References


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