RADIALLY SYMMETRIC POLAR ICE SHEET FLOW WITH EVOLVING ANISOTROPIC FABRIC

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Summary A radially symmetric, gravity-driven, steady flow of a grounded polar ice sheet with a prescribed temperature field is considered. The ice is modelled as an incompressible, non-linearly viscous and anisotropic fluid with evolving orthotropic fabric. To describe the evolution of the fabric as an initially isotropic free surface ice descends to depth in an ice sheet, a constitutive law relating the deviatoric stress to the strain-rate and strain is applied. The solution is constructed as a leading order approximation derived from asymptotic expansions in a small parameter that reflects the small ratio of stress and velocity gradients in the longitudinal direction to those in the thickness direction. Results of calculations show the effects of a bed topography on the ice sheet thickness profile and the velocity components. Additionally, the influence of the temperature field and the free surface snow accumulation rates on the flow is illustrated.

EXTENDED SUMMARY

Ice cores retrieved from large polar ice caps in Antarctica and Greenland reveal strong anisotropic fabrics, with individual ice grain c-axes aligned along some preferential directions (Thorsteinsson *et al.*, 1997). These fabrics develop and evolve in the material in its response to changing stress and deformation states which ice experiences during its passage through the depth of an ice sheet. In current numerical models used to simulate the flow of polar ice sheets on geophysical scales the mechanism of evolving (induced) anisotropy is ignored and, for the sake of simplicity of the analysis, ice is treated as an isotropic material. In order to account for local changes in viscous properties of ice, resulting from its anisotropy, the material rheology is commonly described in terms of ad hoc adopted so-called enhancement factors for compression and shear (Budd and Jacka, 1989). In recent years, however, first attempts have been made to incorporate the ice fabric anisotropy in large polar ice sheet models, but so far they have essentially been restricted to the problems in which a flow field is determined for a given (that is observed in the field) fabric (Mangeney *et al.*, 1997). In this work we make a step further and consider a more general problem in which the effect of the flow field itself on the fabric formation and its subsequent evolution is also taken into account, so that the fabric determination constitutes an integral part of the problem solution.

We investigate a steady, radially symmetric, gravity-driven flow of a grounded ice sheet, with sliding conditions on the bedrock. A temperature field within the glacier is prescribed so that the mass and momentum balance equations are uncoupled from the energy balance equation. Ice is assumed to be an incompressible and non-linearly viscous fluid. The rheology of ice is described by an orthotropic constitutive law formulated in Staroszczyk and Morland (2001) and Morland and Staroszczyk (2003). This law, of an additive form to account for isotropic and anisotropic parts of the ice response, expresses the deviatoric stress in terms of the strain-rate, strain and three structure tensors based on the current (evolving) principal stretch axes, and involves two fabric response coefficient functions which determine the strength of the anisotropy. Furthermore, the strong effects of the temperature and the deviatoric stress magnitudes on the isotropic ice viscosity are taken into account.

In view of the complexity of the computations of the full model equations for a grounded ice sheet, the current application is based on a simplified *reduced model* (Morland, 1997) which exploits the large aspect ratio of polar ice masses, an approach known in theoretical glaciology as the 'shallow ice approximation' (Hutter, 1983). Hence, leading order balances are derived from asymptotic expansions in a small parameter (of a typical magnitude between 10^{-3} and 10^{-2}) which reflects the small ratio of the stress gradients and velocity components in the longitudinal direction to those in the thickness direction. The momentum balance equations are integrated through the ice thickness to eliminate one spatial coordinate, and an asymptotic analysis is carried out to determine a unique free surface slope at the sheet margin for a chosen form of a sliding law. Consequently, the problem is reduced to the solution of a second order parabolic differential equation for the ice sheet thickness profile, forming a two-point boundary-value problem to determine the free surface elevation of the ice sheet. In the case of isotropic ice this completes the solution of the problem. However, when the anisotropy of ice is involved, additional equations are required to account for changing properties of ice particles as they move from the free surface of an ice sheet to its depth. Thus, evolution equations of the hyperbolic type for five deformation gradient components involved are solved, adding substantially to the complexity of calculations. Results of numerical simulations show how the ice fabric changes within the ice sheet, from the isotropic fabric in the accumulation zone on the free surface of the glacier to strongly anisotropic fabrics in regions of high compression or shearing. Furthermore, the effects of the bed topography on both the ice sheet thickness profile (particularly on the ice sheet span) and the velocity and stress components are investigated. Also, comparisons for different temperature profiles and different forms of the functions defining the snow accumulation and ablation rates on the free surface of the glacier are carried out.

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