

## Travelling waves and transition to turbulence in pipe flow

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### ABSTRACT

Hagen-Poiseuille flow through a pipe of circular cross section belongs to the class of shear flows that does not become linearly unstable. The situation is similar to Taylor-Couette flow with the inner cylinder at rest as well as to Taylor-Couette in the limit of large radii where the system approaches plane Couette flow [3]. In these cases the transition to turbulence is not related to series of symmetry-breaking linear instabilities but rather with the formation of nonlinear 3-d states that are not connected to the laminar profile and that seem to form a chaotic saddle.

As indicators for this process we present exact coherent states to pipe flow, we discuss the sensitive dependence on initial conditions quantified by the Lyapunov exponent of turbulent trajectories, and the exponential distribution of life times of the turbulent state. All our findings are consistent with the formation of a strange saddle (repellor) in phase space.

A family of three-dimensional **travelling waves** for flow through a straight pipe of circular cross section has been identified (Fig. 1) [1]. They were found by a Newton-Raphson method which was implemented with a spatial resolution of 21 modes in azimuthal and downstream direction and 44 Legendre polynomials radially. This gives us about 8700 dynamically active velocity coefficients. They originate in saddle-node bifurcations at Reynolds numbers as low as 1250, where  $Re$  is based on the mean downstream velocity and the pipe diameter. All states are immediately linearly unstable at the bifurcation.

The travelling waves are dominated by pairs of downstream vortices and streaks (Fig. 1). The high speed streaks near the wall move much less than the low speed streaks closer to the center of the pipe. The dominating structures are streamwise streaks and streamwise vortices that closely resemble coherent states in other shear flows like the wavy-vortex flow in Taylor-Couette or plane Couette flow. The dynamical significance of the exact coherent states is that they provide a skeleton for the formation of a chaotic saddle that can explain the intermittent transition to turbulence and the sensitive dependence on initial conditions in this shear flow [1,2].

For dynamical **lifetime experiments** we follow the experiments of Darbyshire & Mullin [4] and keep the volume flux constant. We extended our numerical investigations to times of 2000 or more (unit of time: mean streamwise velocity/radius), far exceeding the values accessible in the longest currently available laboratory experiment. As initial conditions we used a high amplitude uncorrelated superposition of spectral modes. The spatial structure is so rich and the amplitude so high that the probability to trigger turbulent dynamics is maximal for a wide range of Reynolds numbers. Various conclusions can be drawn from our lifetime experiments: We find a transitional Reynolds number somewhat below  $Re = 2000$ . The minimum amplitude to trigger a long living turbulent dynamics decreases with Reynolds number. We observe strong sensitivity on initial conditions as well as on parameters which results in large fluctuations in turbulent life times (Fig. 2) [2]. The regions of quickly decaying and long-living trajectories are separated by complicated, fuzzy stability borders. The results are in agreement with experiments by Darbyshire & Mullin [4].

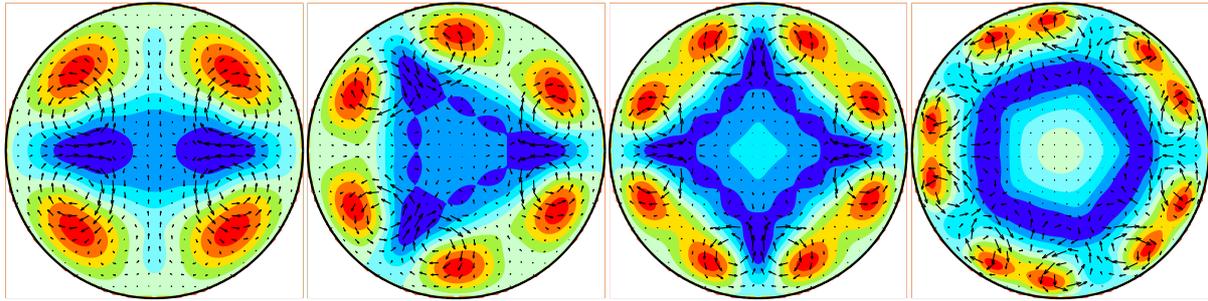


Figure 1: Travelling waves with symmetries  $C_n$  with  $n = 2, 3, 4,$  and  $5$  (invariance under rotation around the pipe axis by an angle  $2\pi/n$ ). In order to highlight the topology of the states all states are averaged in downstream direction. The representation of the velocity field has vectors for the in-plane motion and color coding for the downstream motion.

The strong sensitivity of lifetimes on initial conditions and on parameters calls for a statistical description of the transition process. Therefore, the distribution of turbulent lifetimes has been calculated and it is shown in Fig. 2.

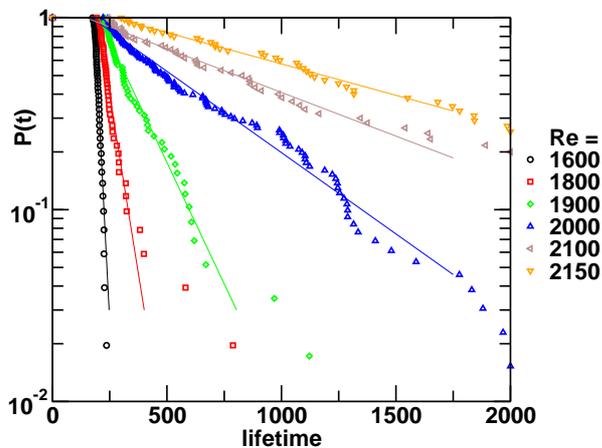


Figure 2: The probability for a single trajectory to still be turbulent after a certain integration time. Between 50 and 100 trajectories have been integrated per Reynolds number.

Although the fluctuations are large an exponential distribution can be identified within the statistical errors. This can be interpreted as a constant rate of escape from the turbulent state which is a major characteristic of a chaotic repeller.

Increasing the Reynolds number up to  $\approx 2200$  the median of the turbulent life times as well as the fluctuations increase rapidly until the median reaches the cut-off life time of 2000. It might even be the case that the escape routes from the repeller are closed and it is turned into a strange attractor.

## REFERENCES

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