XXI ICTAM, 15-21 August 2004, Warsaw, Poland

STRESS-FOCUSING EFFECT IN A SPHERICAL ZIRCONIA INCLUSION WITH DYNAMICALLY TRANSFORMING STRAINS

Toshiaki Hata*
*Faculty of Education, Shizuoka University, Shizuoka, 422-8529, Japan

Summary Some composite materials, such as Zirconia toughened ceramics, are remarkable material, which has a high strength, a high elastic modulus, and an improved toughness, etc. These good qualities are made possible through the stress-induced phase transformation of composite particles, which is accompanied by a volumetric expansion. When a spherical inclusion in infinite elastic domain is suddenly subjected to an instantaneous phase transformation, stress waves occur at the surface of spherical inclusion the moment instantaneous transformation strains are applied. The wave may accumulate at the center of the inclusion and show the stress-focusing effects, even though the initial stress should be relatively small.
This paper analyzes the stress-focusing effect caused by the instantaneous phase transformation in the spherical Zirconia inclusion. By using the ray theory, the numerical results give a clear indication of the mechanism of stress-focusing effect in an inclusion embedded in the infinite elastic me.

INTRODUCTION

Recently, the transformation toughening of ceramics has attracted considerable attention in several works such as Mikata and Nemat-Nasser[1]. The mechanism in the toughening of ceramics is the stress-induced phase transformation of a Zirconia particle as shown in the work by Garvie *et al.*[2], which is accompanied by volumetric expansion. Due to this expansion, the composite material consisting of Zirconia particles within a brittle matrix becomes more resistant to the thermal fracture. While in the dynamic state the mechanism in the toughening of ceramics is not well understood.
In the paper a phenomenological model is proposed to describe the situation, which involves the dynamic inhomogeneity with a stress-induced martensitic transformation in a spherical particle of Zirconia embedded in an infinite elastic matrix.
When an infinite elastic medium with a spherical inclusion of Zirconia is suddenly subjected to a uniform phase transformation, stress waves occur at the interface of a spherical inclusion the moment a phase transformational impact is applied. The stress wave in an inclusion proceeds radially inward to the center of the inclusion. The wave may accumulate at the center and show the stress-focusing effects in the work of Hata[3].
As for the study of the stress-focusing effects, Hata[4] solved, in an exact manner, the thermal stress-focusing effect in a spherical inclusion embedded in an infinite medium by using the ray integrals. In this paper, following the ray methods, we clarify the stress-focusing effect caused by the instantaneous phase transformation in a spherical inclusion of Zirconia. It should be noted that the mechanism in the toughening of ceramics in the steady state does not hold in the dynamic state.

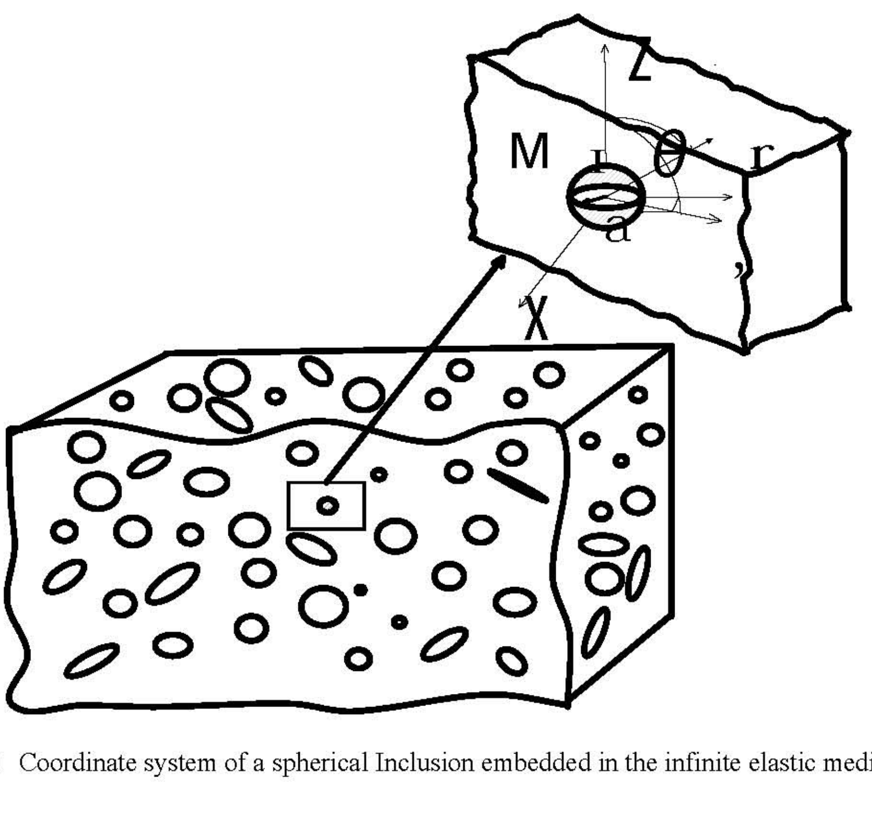

Fig.1 Coordinate system of a spherical Inclusion embedded in the infinite elastic medium

Formulation of Problem

The geometry of the problem is shown in Fig.1. The medium and the inhomogeneity are denoted M and I, respectively. Consider an infinite isotropic elastic medium of M containing a spherical Inclusion of I with an eigenstrain (or transformation strain) $e^*_{ij}(\in \Omega)(i,j=1,2,3)$. The governing equations for an inclusion are given by Mura[5] as

$$\sigma^I_{ij,j} = \rho_{0I}\ddot{u}^I_i, \quad \sigma^I_{ij} = C^I_{ijkl}(e^I_{kl} - e^*_{kl}), \quad e^I_{kl} = (u^I_{k,l} + u^I_{l,k})/2, \tag{1}$$

where ρ_{0I} is the mass density and C^I_{ijkl} is the elastic tensor of the inclusion as follows;

$$C^I_{ijkl} = \mu_I(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) + \lambda_I\delta_{ij}\delta_{kl} \tag{2}$$

For the medium with a spherical inclusion, the boundary conditions on the interface of $r=a$ are

$$\sigma^I_r = \sigma^M_r, \qquad u^I_r = u^M_r. \tag{3}$$

The additional condition is that the displacement of an infinite medium at infinity is $u^M_r = 0$. The medium with a spherical inclusion is at rest prior to time $t=0$ and the initial conditions of displacement are

$$u^I_r(r,t) = u^I_r(r,t)_{,t} = 0, \qquad u^M_r(r,t) = u^M_r(r,t)_{,t} = 0. \tag{4}$$

In the analysis the eigen strains of phase transformation are given by

$$\begin{aligned} &e^*_r = \epsilon_0 f(t) \quad e^*_\theta = e^*_\varphi = \epsilon_0 f(t) \qquad (0 < r \le r_0) \\ &e^*_r = e^*_\theta = e^*_\varphi = 0 \qquad (r_0 < r \le a) \end{aligned} \tag{5}$$

Under the action of phase transformation, the eigen strains expand along its radial direction self-similarly with a constant velocity V. Therefore, the function $f(t)$ is given by

$$f(t) = H[t - \frac{r}{V}] \tag{6}$$

In order to analyse the wave propagation we introduce the equation of motion with the time-dependent eigen stains. Applying Laplace transform, we can easily obtain the Laplace transformed solution. Since it is difficult to get the inverse Laplace transformed solution, we introduce the ray theory. By using the theory, the Laplace transformed solutions of stress waves in a spherical inclusion and in the infinite medium are sorted out into rays according to the ray path of multiply reflected waves. The inverse Laplace transform of each ray leads to the exact solution of the transient response up to the arrival time of the next ray. Following the ray methods, we clarify the stress-focusing effect in a spherical Zirconia inclusion caused by the moving phase transformation and carried out the numerical calculation concerned with the strength of the Zirconia composite[6].

CONCLUSIONS

The major accomplishment of this study has been in gaining a better understanding of the stress-focusing effect in a spherical Zirconia inclusion with the instantaneous phase transformation embedded in an infinite ceramics matrix. The stress-focusing effects in a spherical inclusion occur when stress waves reflect from the spherical surface of the inclusion and accumulate radially inward to the center. It should be important to note that the singular behavior of both stresses and displacement can be observed at the center of an inclusion for the focusing time in the case of instantaneous phase transformation.

Following the ray methods, we clarify that the stress-focusing effects in a spherical inclusion play an important role. Therefore it should be noted that the stress-induced mechanism of phase transformation in the toughening of the Aluminum Oxide ceramics with a Zirconia inclusion in the steady state does not hold in the dynamic state.

References

[1] Mikata, Y.and Nemat-Nasser, S.: Elastic Field Due to a Dynamically Transforming Spherical Inclusion, J. *Applied Mechanics*, **57:** 845-849,1990

[2] Garvie, R.C., Hannink,R.H. and Pascoe,R.T. Ceramic Steel, *Nature*, **258:**703-704,1975

[3] Hata, T.: Thermal Stress-Focusing Effect Following Rapid Uniform Heating of Spheres and Long Cylindrical Rods, *Journal of Thermal Stresses*, **20:** 819-852,1997

[4] Hata, T.: Analysis of the Thermal Stress-Focusing Effect in a Spherical Inclusion Embedded in an Infinite Elastic Medium, *JSME International Journal*,**42:** 176-182,1999

[5] Mura, T.: Micromechanics of defects in solids, Mura, Martinus Nijhoff Publishers, Dordrecht, The Netherlands, second edition,1987

[6] Lange, F.F.: Transformation toughening Part 4 Fabrication, fracture toughness and strength of Al_2O_3-ZrO_2, *J. Materials Science*,**17:** 247-254,1982