SIZE EFFECT IN TENSILE FRACTURE OF CONCRETE - A STUDY BASED ON LATTICE MODEL APPLIED TO CT - SPECIMEN

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Summary: In this paper, a study on size effect in tensile fracture of concrete and simulation of strain softening of plain concrete using 2D lattice model is presented. A compact tension specimen (CT) is adopted and triangular lattice network is used to simulate the heterogeneous structure of concrete. A computer program is used to generate random variation of Young's modulus values for each individual element present in the lattice, which can represent a heterogeneous concrete medium. Load Vs. CMOD plots thus obtained show-softening behavior of concrete. A size effect curve in tensile strength of concrete has been plotted.

Introduction

The phenomenon of cracking is the most important feature of the concrete behavior. When the tensile strength of a material is reached in a structure, cracking will occur. The fracture mechanics studies deals with the conditions around and in front of a crack tip. The fracture of a heterogeneous solid subjected to external displacement is of great importance. Some materials, which are heterogeneous at the meso scale, exhibit strain softening behavior and fail progressively by damage-localization and fracture takes place in their components. Examples of these materials are mortar, concrete, ceramic composites, fibrous materials, rocks. There is similarity in the global behavior of these materials. One of the main characteristics of quasi-brittle materials like concrete and rock, composites are their high heterogeneity. Cracks generally propagate in a direction which is perpendicular to the maximum tensile stress. In heterogeneous materials cracks also follow the weakest links in the material. The crack patterns in this kind of material will therefore never be straight and continuous. Due to the heterogeneity of the material cracks are tortuous and there are overlaps or so called crack face bridges. Nowadays numerical simulation, coupled with theory and experiment, is considered to be an extremely important tool for further material development. In the concrete world, 'numerical concrete', developed by Wittmann and co-workers has already shown encouraging results. E.Schangen [4] *et al.* showed that by means of the generated material structure. Sadouki *et al.* investigated the possibility of analysis numerically to examine the mechanical behavior of concrete, using a particle model. Lattice models are one of the numerical methods developed by researchers for solving the classical problems. The concept of the lattice model is discretization of the continuum by line elements such as bar elements and beam elements, which can transfer forces and moments. Lattice with truss elements corresponds to the classical continuum, while lattice with beam elements corresponds to the micro polar continuum. The heterogeneity of the material can be incorporated in a lattice model, by assigning different strength and or stiffness values to the individual lattice members. By using these lattice model techniques it is possible to identify micro cracking, crack branching, crack tortuosity and bridging, which leads to the fracture process to be followed until complete failure. The concept of lattice analogy can be extended to two-dimensional continua. Essentially, lattice analogy replaces the continuous surface structure by an idealized discrete system, the elements of which are interconnected only at finite nodal or joint points. In such a substitute structure, having its size of subdivisions decreased, the state of stress approaches that of the corresponding continuum. The lattice analogy cannot be completely equivalent to the replaced continuum, an accuracy sufficient for most practical purposes, can be achieved. Over the last 45 years lattice models have been used for various applications. The earliest model was proposed by Hrennikoff [1], who introduced a regular triangular lattice of truss elements to solve classical problems in elasticity. Burt and Dougil *et al.* model a progressive failure in heterogeneous material. Theoretical physicists from the field of statistical mechanics Hans J. Herrman, Alex Hansen and Stephen Roux [2] simulated the brittle failure of disordered materials and the conductivity of the materials using a square lattice with beam elements. Schlangen and Van Mier [5] simulated the fracture in concrete materials with triangular lattice with beam elements. Schlangen *et al.* showed that local rotations at the nodes play an important role in the simulated fracture process. Schlangen and Van Mier *et al.* presented a report on experimental and numerical analysis of micro mechanisms of fracture of cement based composites and simulated a concrete plate with a notch using beam elements. Herrman *et al.* proposed a triangular lattice with elements of random length. In this paper the disorder of the medium is implemented by assigning randomly chosen material properties to the elements in the lattice and graphs related size effect in tensile fracture was obtained using bar element and beam element.

Lattice modeling of Concrete

Concrete is a porous composite material consisting of sand and coarse aggregates embedded in a cement paste matrix. The pores in the material play an essential role in strength, ductility and durability. Researchers in the field of fracture mechanics of concrete modeled concrete in three level approaches. These are micro, meso and macro levels. The meso - level considers the composite nature of concrete and distinguishes between hardened cement paste, aggregate, and a bond layer between cement paste and aggregate constituents. The stress-strain behavior under tensile loading of the aggregate, bond and matrix are assumed to be linear elastic. Disorder must be taken into account when a heterogeneous material like concrete is simulated. In this study a regular triangular lattice is adopted. All lattice member are ascribed material properties of aggregate(gravel), matrix and bond following an order of matrix-bond-aggregate-bond-matrix-bond-aggregate….as shown in fig.2a. In fig.2b. “A” represents the aggregate phase, “M” the matrix and “B” the bond between matrix and aggregate. Material properties, which must be assigned for the three phases are the Young’s moduli and tensile strengths. fig.2a shows a concrete plate with a notch at one of its sides idealized as a lattice with 317 members and 122 joints.

Material Constants

Phase	E (Mpa)	σ_t(Mpa)
Aggregate (A)	70000	10.00
Bond (B)	25000	5.00
Matrix (M)	25000	1.25

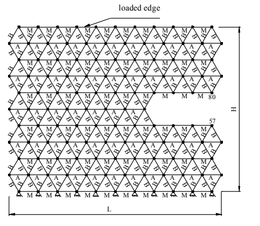


Fig. 2a. Lattice network of a concrete plate with a notch at one of its sides.

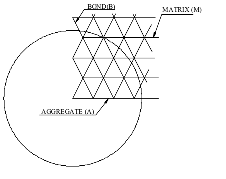


Fig. 2b. Triangular lattice and assigning different properties to the lattice elements

Method of analysis

Crack propagation in the structures is often simulated with finite element methods based on continuum. A standard Finite Element Method program for analysis of lattice structure has been developed. Fracture is simulated by performing a linear elastic analysis of the lattice and removing an element from the mesh that exceeds maximum tensile strength of the element. The displacement control method was adopted for loading the structure. In the present study analysis was carried on a model consisted of regular triangular lattice with bar type of elements. The lattice consists two node bar element which has two degrees of freedom at each node., i.e., U_x and U_y . For displacement simulation, the prescribed displacement 0.001 mm/step is imposed as a constraint. In each step of the fracture analysis the lattice mesh is loaded by external prescribed displacements. The displacement rate 0.001 mm/iteration is chosen which stipulate that the peak load in a displacement-controlled test. Gussian Elimination method which is a direct method is used for the solution of the simultaneous equations. A flow-chart of the implementation procedure is prepared. The x- direction displacements of the nodes located along the model’s upper edge are kept at zero value and the all displacements at the nodes located along the lower edge were suppressed. The upper edge is subjected to a vertical displacement of 0.001mm in Y-direction. The analysis of concrete fracture process consists of a series of elastic steps. Each element stress is calculated and checked against the corresponding limiting stress. The aggregate, bond, and matrix truss elements, whose stresses have reached the limiting values are ascribed a negligible value of modulus of elasticity for corresponding element in the next increment of the load. The material constants and limiting tensile stress values for aggregate, bond and matrix are shown in the above table.

Failure Criteria for lattice members

When performing a fracture analysis on a lattice mesh, a failure criterion has to be chosen. Different criteria for fracture have been adopted and can be found in the literature[2]. A powerful, widely used approach in finite element analysis of concrete is the concept of smeared cracking, introduced by Rashid. According to this concept, the stress in a finite element is limited by the tensile strength of the material, $f_t^{'}$. After the limiting value of strength is reached, the stress in the lattice member decreases. The tensile strength of the three phases is used as fracture parameter. As mentioned introduction Herrmann et al[2]. simulated the fracture of metals using a square lattice and assumed that each individual elements in the lattice fail according to the following criterion motivated by Von Mises’ failure criterion

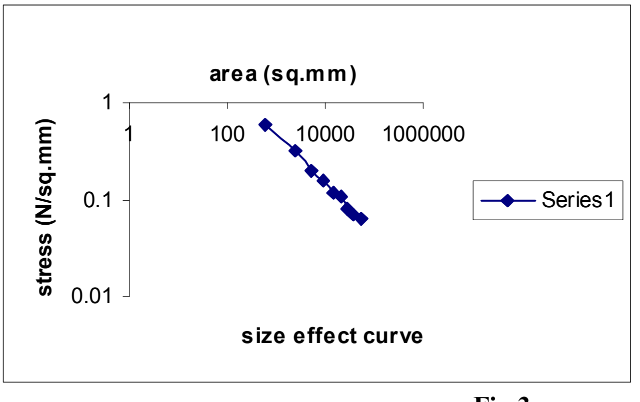
$$p = \left(\frac{f}{t_f}\right)^2 + \frac{\max(|m_1|,|m_2|)}{t_m}$$

Where p is the limiting value that can be related to the probability of failure, f is the axial force in the element, m_1 and m_2 are the end moments and t_f and t_m are the corresponding threshold values, which depend on the given material. In this study whenever the axial stress in a member in lattice model with bar elements reaches limiting stress the modulus of elasticity of that member was reduced to negligible value for the next step. The limit stresses rather than the limit strains have been assumed

Size Effect

After Galilio, Licester *et al* seems to have been the first to investigate the effect of size on the strength of structures made of metals, timber and concrete. The effect of structural size on its normal strength has a long history . In order to illustrate the size dependence in a simple and dimensionless way, Hillerborg[2] introduced the concept of a characteristic length. As a unique material property, The characteristic length $l_{ch} = \frac{EG_f}{f_t^2}$ expresses the fracture of concrete and concrete-like materials, where l_{ch} value is proportional to f_t^2 . This means that brittleness increases with an increase in the strength of concrete, but it decreases with high fracture energy, according to fictitious crack model (F.C.M.). The brittle response of the concrete as a material can not be confused with the brittleness of a concrete structure. The brittleness of concrete structural elements depends on their size. In this way, for the same material, small elements fail with a ductile response, whereas large elements fail in a brittle manner. The variation of the structural response as the size of the structure changes is known as ductile-brittle transition. Bazant's size effect law [2] gives a measure of the brittleness of concrete members. According to this law, $\sigma_N = Bf_t(1+\beta)^{0.5}$ with $\beta = \frac{d}{d_0}$, σ_N = nominal strength, d= structural size, f_t =tensile strength of concrete, B and d_0 =constants. Both d_0 and B depend on the fracture properties of the material and on the geometry (shape) of the structure, but not on the structure size. The size effect is understood as the dependence of the structure strength on the structure size. The strength is conventionally defined as the value of the so called nominal stress is a load parameter defined to the proportional to the load divided by a typical cross sectional area.

Discussion of results



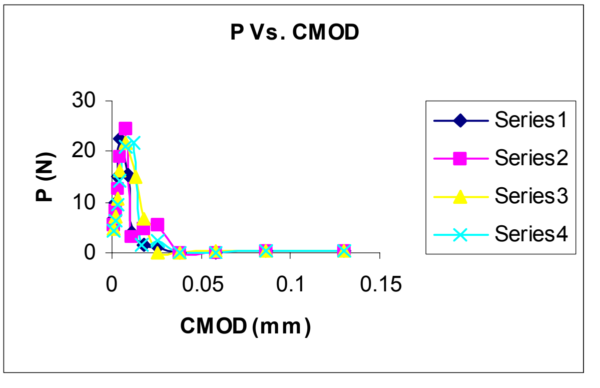


Fig.3

Conclusions

A lattice model at meso level, with a simple linear elastic purely brittle failure law in tension was used for simulating the fracture process of a notched concrete plate subjected to uniaxial tension. The results obtained from simulations with lattice models, depend on the fracture criterion used and the element selection for modeling the lattice. From the graph shown in fig.(3) one can observe that the peak stress in the graph of the applied stress against crack opening displacement of a loaded concrete plate with a notch decreases with increase in the size of the structure.

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