NONLINEAR WAVES IN SHOCK-LOADED SOLIDS

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Summary Nonlinear wave processes in shock-loaded elastic-plastic solids are modeled on the basis of the approximate equations proposed. It is shown that the equations correctly describe the stress-distribution evolution in both the elastic-flow and plastic-flow regions and can be used to solve 1D and 2D problems of pulsed deformation and fracture of elastoplastic solids.

THE APPROXIMATE EQUATIONS DESCRIBING PROPAGATION OF NONLINEAR LONGITUDINAL WAVES IN ELASTOPLASTIC SOLID

Studying the evolution of nonlinear waves generated by a shock loading of solids is of scientific and practical interest. Experiments of this kind are usually performed under conditions corresponding to the stress range from several gigapascals to tens of gigapascals. In these cases, the stress amplitude is small in comparison with the bulk modulus but exceeds considerably the elastic limit for most metals. Therefore, a set of small parameters can be defined and the asymptotic methods known in the general theory of nonlinear waves may be extended to these problems.
As a result a system of approximate independent nonlinear equations belonging to different directions of longitudinal characteristics is obtained. The equations have the form [1, 2]

$$\lambda_i \frac{\partial V_i}{\partial z} - \frac{1}{4}(\alpha+2)V_i \frac{\partial V_i}{\partial \xi_i} - 3\nu \frac{\partial \psi_i}{\partial \xi_i} - \frac{1}{2}\mu \frac{\partial^2 V_i}{\partial \xi_i^2} - \frac{1}{2}\varepsilon_\Delta \int_{-\infty}^{\xi_i} \Delta_\perp V_i \, d\xi_i' = 0 \tag{1}$$

Here $V_i = -\sigma_{11}' + \lambda_i u_1' = O(\varepsilon)$, $\lambda_{1,2} = \pm 1$, $\xi_i = t' - \lambda_i^{-1} x_1'$, $z = x_1'(1 + O(\varepsilon))$, $\Delta_\perp = \partial^2/\partial r'^2 + \partial / r' \partial r'$ $(r' = \sqrt{x_2^2 + x_3^2}/r_0)$, $t' = t/t_0$, $x_1' = x_1/C_0 t_0$; $\sigma_{11}' = \sigma_{11}/K$ and $u_1' = u_1/C_0$ are the components of the stress tensor and velocity vector (K is the bulk modulus, $C_0 = \sqrt{K/\rho_0}$); ψ makes sense of the dimensionless shear stress; t_0 is the characteristic load duration; r_0 is the characteristic size of the loading area. The parameter α is determined from an equation of state. Set of small parameters includes the following: $\varepsilon = P_{max}/K$, $\varepsilon_\Delta = (C_0 t_0 / r_0)^2$ characterizing the transverse divergence of the wave, μ characterizing the internal-friction viscosity and thermal conductivity, $\nu = 2G/K$ (G is the shear modulus) meaning that the stress-deviator components are assumed to be quantities of a higher order of smallness compared to the average stress. Indeed for stresses that occur in typical shock-wave tests for metals $\varepsilon \sim 0.1$ while for the dimensionless components of stress deviator an estimate gives $|s_{ij}'| = \nu \cdot 3|s_{ij}|/(2G) \le \nu \cdot Y/G \sim 0.01 \div 0.001$, where Y is the yield stress.
The equations (1) are closed by a constitutive equation taking into account the elastic-plastic deformation kinetics of solid. For example, the constitutive equation corresponding to the elastoplastic medium of the Prandtle-Reuss type with the Mises yield criterion has the form

$$\frac{\partial \psi_i}{\partial \xi_i} = \frac{1}{3}\frac{\partial V_i}{\partial \xi_i} \text{ for } |\psi_i| \le \frac{Y}{3G} \text{ and } \psi_i = \frac{Y}{3G}\operatorname{sgn}(\psi_i) \text{ for } |\psi_i| > \frac{Y}{3G}. \tag{2}$$

One can see that the nonlinear waves belonging to different directions of longitudinal characteristics are governed separately by systems consisting of the transport equation (1) and the constitutive equation of solid. The interaction of these (oppositely facing) waves can be described implicitly by nonuniform deformation of the phase variables in the solution constructed without regard to the interaction. To describe the interaction, the corrections of order ε and ν are introduced into the phase variables

$$\xi_i = t' - \lambda_i^{-1}(x_1' + \varepsilon \Phi_i(x_1', t') + \nu \theta_i(x_1', t')),\ i = 1,2\,. \tag{3}$$

Equations for phase functions Φ_i and θ_i taking into account a change of a phase velocity caused by the square-law nonlinearity and the elastoplasticity can be written in the form

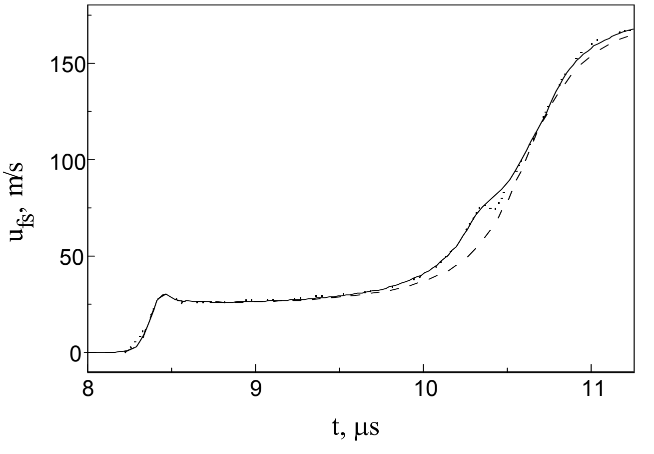
$$\frac{\partial(\varepsilon\Phi_i + \nu\theta_i)}{\partial z} = -\frac{\left(C_\sigma^2\right)\big|_{V_1+V_2} - \left(C_\sigma^2\right)\big|_{V_i}}{2C_0^2}, \tag{4}$$

where $C_\sigma = (dz/dt')_V$ is the Lagrangian phase velocity associated with the stress profile of longitudinal wave. This quantity has clear physical sense and can be measured in experiment (see, for example, [3]).

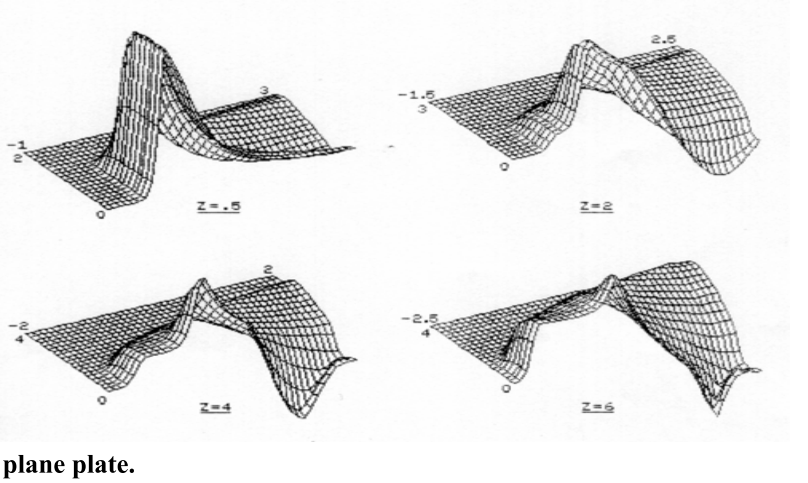
The equations (1)-(4) are used to solve one- and two-dimensional problems of propagation and interaction of shock waves.

PROBLEMS OF PROPAGATION AND INTERACTION OF SHOCK WAVES

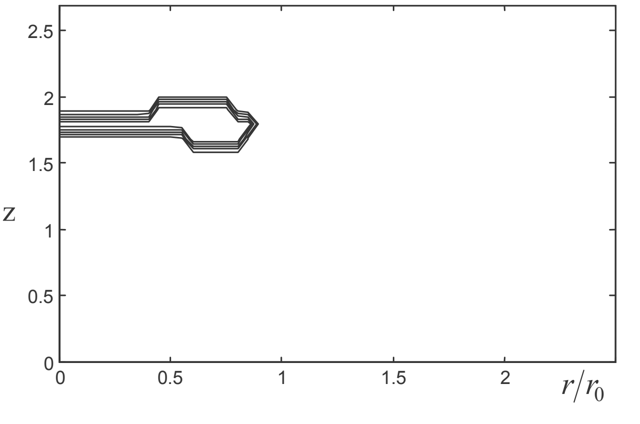
Reflection of shock wave from a free surface of plate. Simulation of the Taylor and Rice experimental data [4, p. 29] for 50.8 mm armco-iron plate loaded by 170 m/s direct impact is conducted using Eqs. (1), (3), (4) and Gilman's constitutive model.
Fig 1. Velocity of the free surface versus time: the solid curve refers to the numerical solution taking into account interaction between the incident and reflected waves, dashed curve refers to the numerical solution ignoring this interaction, and the dotted curve refers to the experimental data. One can see that the solution that takes into account the interaction agrees well with the experimental data.



Propagation of a two-dimensional shock wave in an elastoplastic half-space. A shock pulse is produced by detonation of a cylindrical tablet of explosive located on the half-space. The ratio of thickness of the tablet to its diameter is equal to 0,2. Material of the half-space is the steel.
Fig. 2 shows snapshots of the wave (the numerical solution) at different distances from surface. The elastic precursor and elastic unloading are clearly visible. In general one can see that the approximate equations correctly describe both the plastic flow and elastic flow regions.



Simulation of a two-dimensional damage of a plane plate. Using Eqs. (1) and (2), we solve numerically the problem of the normal-impact damage of a plane plate, produced by a cylindrical impactor with a velocity of 185 m/sec. Material of the plates is aluminum. The impactor thickness is l=1.14 mm and target thickness is L=2.8·l. The radius of the impactor is r_0= 6·l. Equations governing the evolution of the material damage are taken the same as in [5].
Fig. 3 shows snapshot of the right-hand half of the plate cross-section for t=0.856 μsec (the impact is performed along the lower surface). Contour lines correspond to the specific volume of voids greater than 0,01. The central region enclosed by the lines corresponds to the failed material. One can see that the impact failure forms a disc-like crack.



References

[1] Myagkov N.N.: Nonlinear waves in shock-loaded condensed matter. *J. Phys. D: Appl. Phys.* **27:** 1678-1686, 1994.
[2] Myagkov N.N.: Asymptotic modeling of nonlinear wave processes in shock-loaded elastic-plastic solids. *J. Appl. Mech. Techn. Phys*. **44**: 249-254, 2003.
[3] Asay J.R.,.Chhabildas L.C., Dandekar D.P.: Shear strength of shock-loaded polycrystalline tungsten. *J. Appl. Phys.* **51**: 4774-4783, 1980.
[4] Morris C.E.(ed): Los Alamos Shock Wave Profile Data. University of California, Berkeley 1982.
[5] Seaman L., Curran D.R., Shockey D.: Computational models for ductile and brittle fracture. *J. Appl. Phys*. **47:** 4814-4826, 1976.