SCALING LAWS FOR THERMAL CONVECTIONS

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Summary Recently, there has been much progress in deriving comprehensively the scaling laws for Rayleigh-Bénard convections. We show here that the relatively intuitive and simple model of Castaing et al, with minor modifications, can give many of these 'new' scaling laws as well. We also apply this method to convections in an unsealed enclosure (i.e. one with an inlet and an outlet) to obtain the scaling laws for different convective regimes.

THE CASTAING ET AL MODEL REVISITED

For Rayleigh-Bénard convections obeying the Boussinesq approximation, both the Reynolds number Re (for the large scale velocity) and the Nusselt number Nu are functions of Rayleigh number Ra and Prandtl number Pr. Specifically, there is strong evidence from various theories and experiments to suggest that the scaling laws $Nu \sim Ra^{\gamma}$ and $Re \sim Ra^{\beta}$ exist. The models presented here mainly concern calculating the values of the real number exponents γ and β.
Castaing et al [1] devised a model to explain their experimentally measured $\gamma = 0.282 \pm 0.006$. (N.B. Here, we have simplified the derivations as we are less concerned with other scalings. We have also adapted it to account for the effect of Prandtl number).
Three flow regions are identified: fluid breaks from the boundary layer and accelerates in the mixing zone to speed that allows the fluid to merge into the well-mixed central region with a large-scale stirring velocity and temperature fluctuations (much like a lava lamp, see FIG. 1).

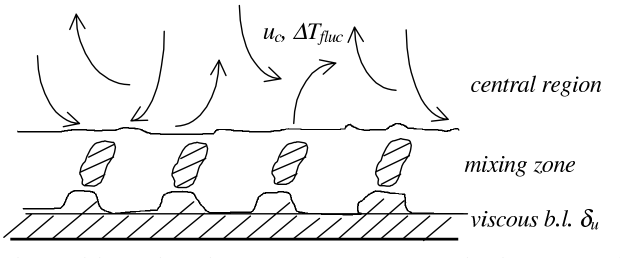


FIG. 1 For $Pr \gtrsim 1$. A stable thermal boundary layer cannot exist outside the viscous boundary layer. Detaching thermal filaments have a length-scale of δ_u.

The method uses mostly well-known relationships involving the large scale velocity, heat flux, both the thermal and kinetic boundary layer thicknesses, the dimensionless numbers, and the balancing of the buoyancy force and the viscous force where the filaments have accelerated to sufficiently high velocity to join the central region. The result from combining these relationships are $Nu_{(Pr \gtrsim 1)} \sim Ra^{2/7}$ and $Re_{(Pr \gtrsim 1)} \sim Ra^{3/7} Pr^{-2/3}$. We then apply the technique for the $Pr \lesssim 1$ case and derive $Nu_{Pr \lesssim 1} \sim Ra^{2/7} Pr^{2/7}$ and $Re_{(Pr \lesssim 1)} \sim Ra^{4/7} Pr^{-3/7}$. Results from more sophiticated theories and recent experiments (e.g. [2,3]) are used for comparisons.
As Grossmann & Lohse [2] have shown theoretically, and Niemela & Sreenivasan [4] with experiments, this is not the end of the story. We show that the Castaing et al model with slight modifications can produce a range of other scalings (and suggesting what processes or assumptions are involved) including $Nu_{(Pr \gtrsim 1)} \sim Ra^{1/4} Pr^{-1/12}$, $Re_{(Pr \gtrsim 1)} \sim Ra^{1/2} Pr^{-5/6}$, $Nu_{(Pr \lesssim 1)} \sim Ra^{1/5} Pr^{1/5}$, $Re_{(Pr \lesssim 1)} \sim Ra^{2/5}$, and even the classical so-called ultimate regime $Nu_{(Pr \lesssim 1)} \sim Ra^{1/2} Pr^{1/2}$. The last scaling, incidentally, agrees with Grossmann & Lohse's suggestion that the ultimate regime may exist only in small enough Prandtl and large enough Rayleigh number.

HEAT TRANSFER IN ENCLOSURE WITH THROUGHFLOW

In this section, we are interested in how forced convections affect the scaling laws. An enclosure similar to that in Rayleigh-Bénard convections, but with inlet and outlet throughflow, is being considered (FIG. 2). We use techniques similar to those described above, the main adaption being that in the forced convections, the inertial force balances the viscous force instead of the balance of buoyancy and viscous forces as in free convections.

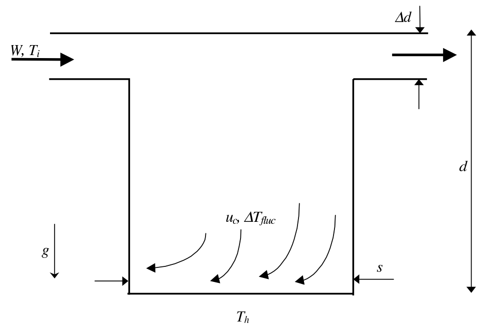
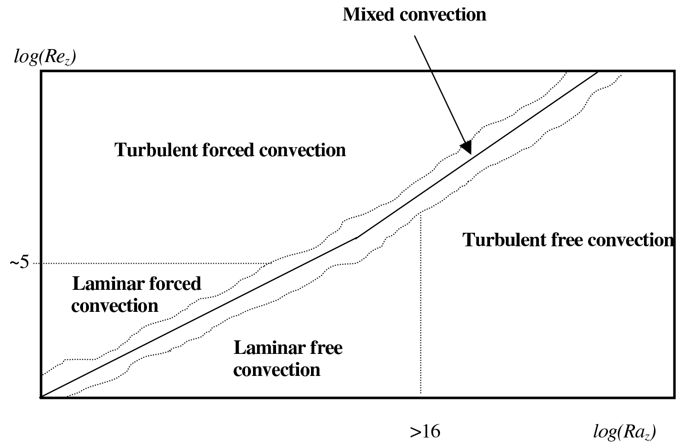


FIG. 2 A stationary cavity with heating from below and throughflow of cooling air with speed W and temperature T_i.

Scaling laws for the regimes of forced convections and turbulent (or laminar) boundary layer, free convections and turbulent (or laminar) boundary layer, and mixed convections are derived. The boundaries delineating the different regimes are obtained too. FIG. 3 summarises the results in a Reynolds number vs. Rayleigh number phase diagram.



Correlations for the different regimes:

Turbulent forced convection: $Nu_z \sim Re_z^{3/4}$
$Nu_z \sim Ra_z^{1/3} Pr^{-2/3}$

Laminar forced convection: $Nu_z \sim Re_z^{1/2}$
$Nu_z \sim Ra_z^{1/5}$

Turbulent free convection: $Nu_z \sim Ra_z^{1/2} Pr^{1/2}$
$Nu_z \sim Re_z^{1/2} Pr^{1/3}$

Laminar free convection: $Nu_z \sim Ra_z^{1/4} Pr^{-1/12}$
$Nu_z \sim Re_z^{1/2} Pr^{1/3}$

FIG. 3 Convections regime diagram for enclosure with throughflow. That for the horizontal or vertical flat plate has similar delineations.

Not too surprisingly, the scalings and regimes are similar to those obtained by other methods for flows over a flat plate. The diagram indicates that many laboratory and computational studies are conducted in the laminar free convections and laminar forced convections regimes. The results thus obtained are combinations of $\gamma = 1/4$ and $\gamma = 1/5$ scalings.

References

[1] Castaing, B., Gunaratne, G., Heslot, F., Kadanoff, L., Libchaber, A., Thomae, S., Wu, X–Z., Zaleski, S. & Zanetti, G., Scaling of hard thermal turbulence in Rayleig–Bénard convection, *J. Fluid Mech.*, **204**:1–30, 1989.
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