TIME-OPTIMAL CONTROL OF HYDRAULIC MANIPULATORS

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Summary The paper presents a control method, which makes a hydraulic arm robustly track a desired joint space path at high speed. Optimisation for speed is performed for the worst expected dynamic condition. The reference trajectory is given as a parametric function in a manipulator's joint space. Portions of the trajectory are online shrunk or stretched in time, to fully utilize capacity of the actuators.

INTRODUCTION

Many hydraulic manipulators work in unstructured surroundings at transporting loads, whose mass is not precisely known. In outdoor locations it is likely, that the desired path of the arm is input interactively by a human operator. The purpose of automatic control is then to follow the received trajectory closely, despite uncertainties in the arm's dynamics. Since, in many cases, high speed of tracking does not affect quality of performance, it is also desirable to speed up tracking to save time. Much work has been done into optimising for speed joint space trajectories of general manipulators. Examples include [1], where optimisation of open-loop tracking control is performed, or [2], where a learning algorithm is used to speed up execution of repetitive tasks of industrial manipulators. In this paper an algorithm is presented, intended for hydraulic manipulators performing non-repetitive tasks. It takes into account feedback effects of control, and uncertainties in the manipulator's equations of motion. An arbitrary reference trajectory is given, as a sequence of desired manipulator's configurations. The actual timing of the trajectory is then chosen such, that it fully utilizes the capacity of actuators, in the most disadvantageous dynamical case within uncertainty bounds.

EQUATIONS OF MOTION

Hydraulic manipulators are open chains of links, connected by prismatic or revolute joints. Each joint is actuated by a linear or rotary hydraulic motor. Assuming rigidity of links and joints and neglecting friction, dynamics of an arm, under the action of motor forces, is described by Lagrange equations of motion

$$\mathbf{D}\,\ddot{\mathbf{x}} + \mathbf{H} = \mathbf{F} \tag{1}$$

where $\mathbf{x}$ is a vector of generalized coordinates of the arm (chosen here as piston displacements to simplify notation), $\mathbf{D}(\mathbf{x})$ is a positive definite inertia matrix, $\mathbf{H}(\mathbf{x},\dot{\mathbf{x}})$ is a vector of gravitation, Coriolis and centrifugal forces, and $\mathbf{F}$ is a vector of forces exerted by hydraulic motors. It is further assumed, that each motor acts independently (has its own hydraulic fluid supply). Fluid flow to and from hydraulic motors is governed by servovalves, whose spool displacements are the control inputs to the system. It is assumed, that flow intensities across valves can be accurately modelled by the square-root law $Q = C\sqrt{\Delta P}\,u$, where Q is the flow intensity, C is a valve dependent flow coefficient, ΔP is the pressure drop across the valve, and u is the spool displacement. The relation between piston forces and spool displacements is then given by

$$\dot{\mathbf{F}} = \mathbf{f}\,\mathbf{u} + \mathbf{g} \tag{2}$$

where $\mathbf{u}$ is the vector of all spool displacements, diagonal and positive definite matrix $\mathbf{f}(\mathbf{P}_1,\mathbf{P}_2,\mathbf{x})$ and vector $\mathbf{g}(\mathbf{P}_1,\mathbf{P}_2,\mathbf{x},\dot{\mathbf{x}})$ represent properties of the hydraulic system, and $\mathbf{P}_1$, $\mathbf{P}_2$ are vectors of pressures in motor chambers. Differentiating equation (1) and substituting equation (2) yields

$$\dddot{\mathbf{x}} = \mathbf{D}^{-1}\mathbf{f}\,\mathbf{u} + \mathbf{R} \tag{3}$$

where $\mathbf{R}(\mathbf{P}_1,\mathbf{P}_2,\mathbf{x},\dot{\mathbf{x}},\ddot{\mathbf{x}})$ collects all remaining terms. Equation (3) is of third order, because of the inclusion of hydraulic system's dynamics. It forms the basis for further control study.

CONTROL OUTLINE

Problem formulation

The reference trajectory is given as a parametric curve $\mathbf{x}_\mathbf{r}(s)$ in a manipulator's joint space, where s is an arbitrary parameter. For the purposes of control it is unimportant, what is the particular choice of parameter. However it is assumed, that $\mathbf{x}_\mathbf{r}(s)$ is sufficiently smooth in s, and takes only physically feasible values. Equations of motion (3) are not exactly known in the sense, that **D**, **f** and **R** might really differ from their nominal values. In particular, variations of **D** are mostly introduced by loads transported by the manipulator. However, all uncertainties are bounded, and values of the bounds are known. Also, variations in values of parameters do not change the general positive definiteness properties of the matrices. There are also bounds on admissible values of spool displacements $\underline{u}_i \le u_i \le \overline{u}_i$, and motor chamber pressures cannot exceed pump and reservoir pressures. The purpose of control is to make the arm traverse $\mathbf{x_r}(s)$ at the highest possible speed, while guaranteeing robustness of motion to uncertainties. The problem can be decomposed into two stages: first choosing a robust tracking control, then optimising the reference trajectory for speed, with respect to the chosen tracking strategy.

Robust tracking control

Many techniques can be used to assure robust trajectory tracking. Here a discontinuous control technique, with a boundary layer to smooth the control signal, is adopted. By expanding equation (3) into the traditional first order state space form and using Lyapunov analysis, robust control can be derived of the general form

(4)
$$\mathbf{u} = -\alpha\,\bar{\mathbf{f}}^{-1}\bar{\mathbf{D}}\frac{\mathbf{v}}{\|\mathbf{v}\|} \quad \textit{for} \quad \|\mathbf{v}\| \geq \varepsilon$$
$$\mathbf{u} = -\alpha\,\bar{\mathbf{f}}^{-1}\bar{\mathbf{D}}\frac{\mathbf{v}}{\varepsilon} \quad \textit{for} \quad \|\mathbf{v}\| < \varepsilon$$

(5)
$$\alpha \geq \|\mathbf{v}\|\frac{\mathbf{v}^{\mathrm{T}}(\mathbf{\Omega} - \dddot{\mathbf{x}}_\mathbf{r})}{\mathbf{v}^{\mathrm{T}}\mathbf{D}^{-1}\mathbf{f}\,\bar{\mathbf{f}}^{-1}\bar{\mathbf{D}}\,\mathbf{v}}$$

where $\bar{\mathbf{f}}(\mathbf{P_1},\mathbf{P_2},\mathbf{x})$ and $\bar{\mathbf{D}}(\mathbf{x})$ are nominal values of $\mathbf{f}$ and $\mathbf{D}$ respectively, $\mathbf{\Omega}(\mathbf{P_1},\mathbf{P_2},\mathbf{x},\dot{\mathbf{x}},\ddot{\mathbf{x}},\mathbf{x_r},\dot{\mathbf{x}}_\mathbf{r},\ddot{\mathbf{x}}_\mathbf{r})$ stands for all terms independent of $\dddot{\mathbf{x}}_\mathbf{r}$, $\mathbf{v}(\mathbf{x},\dot{\mathbf{x}},\ddot{\mathbf{x}},\mathbf{x_r},\dot{\mathbf{x}}_\mathbf{r},\ddot{\mathbf{x}}_\mathbf{r})$ represents the position of the arm in state space with respect to chosen sliding surfaces, ε is the width of the boundary layer, and $\dot{\mathbf{x}}_\mathbf{r}$, $\ddot{\mathbf{x}}_\mathbf{r}$, $\dddot{\mathbf{x}}_\mathbf{r}$ denote derivatives with respect to time *t*. The value of α in (5) is calculated, using the most disadvantageous possible dynamical case. Control (4,5) is valid, if $\mathbf{D}^{-1}\mathbf{f}\,\bar{\mathbf{f}}^{-1}\bar{\mathbf{D}}$ can be guaranteed to be always positive definite. Also for the technique to work, $\mathbf{P_1}$, $\mathbf{P_2}$, $\mathbf{x}$, $\dot{\mathbf{x}}$ and $\ddot{\mathbf{x}}$ must be measured in real-time or estimated. Control (4,5) robustly tracks a time dependent reference trajectory to an accuracy, dictated by ε.

Trajectory optimisation

To speed up tracking $s(t)$ is chosen in such a way, that control (4,5) tracking $\mathbf{x_r}(s(t))$, fully utilizes the capacity of actuators. Since $\mathbf{x_r}(s)$ is given explicitly and is differentiable, its successive time derivatives are

(6)
$$\dot{\mathbf{x}}_\mathbf{r} = \frac{d\mathbf{x_r}}{ds}\dot{s} \qquad \ddot{\mathbf{x}}_\mathbf{r} = \frac{d^2\mathbf{x_r}}{ds^2}\dot{s}^2 + \frac{d\mathbf{x_r}}{ds}\ddot{s} \qquad \dddot{\mathbf{x}}_\mathbf{r} = \frac{d^3\mathbf{x_r}}{ds^3}\dot{s}^3 + 3\frac{d^2\mathbf{x_r}}{ds^2}\dot{s}\,\ddot{s} + \frac{d\mathbf{x_r}}{ds}\dddot{s}$$

If at any instant of time s, $\dot{s}$ and $\ddot{s}$ are given, values of $\mathbf{x_r}$, $\dot{\mathbf{x}}_\mathbf{r}$ and $\ddot{\mathbf{x}}_\mathbf{r}$ can be calculated from equations (6). On the basis of those, and the measured values of $\mathbf{P_1}$, $\mathbf{P_2}$, $\mathbf{x}$, $\dot{\mathbf{x}}$ and $\ddot{\mathbf{x}}$, vector $\bar{\mathbf{f}}^{-1}\bar{\mathbf{D}}\,\mathbf{v}$ in equation (4) can be calculated. Since it determines the direction of control vector **u,** the maximum permissible value of α can also be computed from equation (4). It is the value, which makes $\mathbf{u}$ reach control bounds in the direction of $\bar{\mathbf{f}}^{-1}\bar{\mathbf{D}}\,\mathbf{v}$. Using equations (6), $\dddot{s}$ can now be computed from equation (5), as the greatest value (if it exists) satisfying the inequality

(7)
$$\alpha_{\max} \geq \|\mathbf{v}\|\,\mathbf{v}^{\mathrm{T}}\left(\mathbf{\Omega} - \frac{d^3\mathbf{x_r}}{ds^3}\dot{s}^3 - 3\frac{d^2\mathbf{x_r}}{ds^2}\dot{s}\,\ddot{s} - \frac{d\mathbf{x_r}}{ds}\dddot{s}\right)/\left(\mathbf{v}^{\mathrm{T}}\mathbf{D}^{-1}\mathbf{f}\,\bar{\mathbf{f}}^{-1}\bar{\mathbf{D}}\,\mathbf{v}\right)$$

for the most disadvantageous possible values of matrices. In summary, trajectory optimisation for speed is performed online. It is done by integrating in real-time s, $\dot{s}$ and $\ddot{s}$, and obtaining new values of $\dddot{s}$ from equation (7).

CONCLUSIONS

A control algorithm for hydraulic manipulators is presented, which robustly tracks a given reference trajectory, while fully utilizing the capacity of actuators to maximize speed. It is however computationally expensive, and requires real-time measurements of the full state of the system.

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References

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