Effective Properties of Solids Containing Randomly Distributed Multi-Phase Spherical Particles

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With the ever growing demands for the manufacturing of various innovative particle reinforced composite materials, each tailored for a specific purpose, development of an accurate and unified method for estimations of the effective properties of such materials is inevitable. In this paper, the micromechanical method of homogenization is used to estimate the overall mechanical behaviour of solids containing high concentration of heterogeneous spherical particles. The previous treatments in the literature have failed to account for the long and short range interactions of thickly coated particles rigorously. Based on the extension of Eshelby's equivalent inclusion method (EIM) [1] to multi-inhomogeneities [2], the overall behaviour of solids with periodic distribution of multi-phase interacting particles has been estimated by Shodja and Roumi [3], which involved the Fourier series expansions of the eigenstrains. The present theory is applicable to random distribution of multi-phase particles, where there exist variable thickness interfacial transition zones between the matrix and the core inhomogeneities. To account for the randomness of the reinforcement particles the methodology departs from those presented in [3]. Note that, the interactions between different phases of an isolated multi-inhomogeneity system are inherent in the disturbance strains, [2]. The present paper addresses the complex interaction of many interacting multi-inhomogeneities, a scenario encountered when dealing with non-dilute concentration of multiple coated fiber reinforced composites. For the latter situation, the disturbance strain $\boldsymbol{\varepsilon}^{d(\Omega^{\alpha,\beta})}(\mathbf{x})$ in $\Omega^{\alpha,\beta}$ is given by

$$\boldsymbol{\varepsilon}^{d(\Omega^{\alpha,\beta})}(\mathbf{x}) = \sum_{\gamma\ \left|\mathbf{x}_{o\gamma}-\mathbf{x}_{o\alpha}\right|<r}^{\mathrm{NI}} \left\{ \boldsymbol{\varepsilon}^{(\Omega^{\alpha,\beta})}\left(\mathbf{x};\boldsymbol{\varepsilon}^{*(\Omega^{\gamma,1},\Sigma^{\gamma,1})}\right) + \sum_{j=2}^{NL} \boldsymbol{\varepsilon}^{(\Omega^{\alpha,\beta})}\left(\mathbf{x};\boldsymbol{\varepsilon}^{*(\Omega^{\gamma,j},\Sigma^{\gamma,j})}\right) - \varepsilon^{(\Omega^{\alpha,\beta})}\left(x;\varepsilon^{*(\Omega^{\gamma,j},\Sigma^{\gamma,j-1})}\right)\right\}$$

$$\mathbf{x}\in\Omega^{\alpha,\beta}, \qquad \alpha = 1,2,\ldots,\mathrm{NI}, \quad \beta = 1,2,\ldots,NL$$

where $\mathbf{x}_{o\gamma}$ and $\mathbf{x}_{o\alpha}$ are the coordinates of the centers of the γ-*th* and α-*th* interacting multi-inhomogeneities, respectively. In the above relation, r denotes the radius of the interaction zone for the α-*th* particle, NI stands for the number of inhomogeneities, whose centers fall within the interaction zone, i.e., $\left|\mathbf{x}_{o\gamma}-\mathbf{x}_{o\alpha}\right|<r$, and NL is the number of layers surrounding the γ-*th* inhomogeneity. $\Omega^{i,j}$, $i = 1,2,\ldots,\mathrm{NI}$, $j = 1,2,\ldots,NL$, refers to the j-*th* coating layer of the i-*th* core inhomogeneity, $\Omega^{i,1}$. The region $\Omega^{i,1} = \Sigma^{i,1}$, and $\Sigma^{i,j} = \Omega^{i,j} \bigcup \Sigma^{i,j-1}$, Fig. 1.

Results

For verification the experiment of Amdouni et al. [4] on composites containing spherical particles with very thin coatings is reexamined by the present analysis. The material properties are: $\mathrm{E}^{m} = 2.9\,GPa$, $\nu^{m} = 0.4$; $\mathrm{E}^{f} = 73\,GPa$, $\nu^{f} = 0.2$; $\mathrm{E}^{c} = 10\,GPa$, $\nu^{c} = 0.49$ for matrix, fiber, and coating, respectively. It is seen that the theoretical result accords the experiment, Table 1. Next example considers a composite containing spherical particles with thick coatings, for which $\mu^{f}/\mu^{m} = 25$, $\mu^{c}/\mu^{m} = 5$, and $\nu = 0.3$ for all phases. In this example the volume fractions of particles and coatings are f_{fiber}=10%, $f_{coating}$=20%, respectively, Table 2. Note that in Tables 1 and 2, n indicates the total number of particles in the representative volume element (RVE).

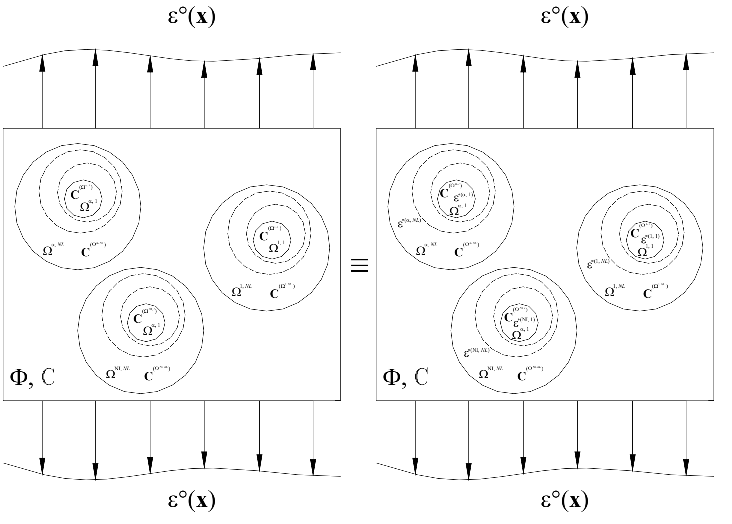

Figure 1. Homogenization scheme for a solid containing non-dilute distribution of multi-phase spherical particles.

n	f_{fiber}	$f_{coating}$	C^*/C_{1212}	C^*/C_{1122}	C^*/C_{1111}	Ps. E*	Exp. E*
25000	10%	1.3%	1.195	1.003	1.063	3.4	3.5
			1.139	1.253	1.219		
			1.205	0.958	1.031		
			1.267	2.014	1.729		

Table 1. Comparison of the elastic moduli: present study (Ps. E*) vs. experiment of Amdouni et, al. [4] (Exp. E*).

n	f_{fiber}	$f_{coating}$	C^*/C_{1212}	k^*/k	C^*/C_{1122}	C^*/C_{1111}
1000	10%	20%	1.216	1.151	1.136	1.165
			1.001	1.320	1.284	1.350
5000			1.216	1.156	1.132	1.178
			1.217	1.155	1.132	1.175
10000			1.278	1.147	1.134	1.158
			1.253	1.148	1.132	1.162

Table 2. Elastic moduli of a composite containing thick coated spherical particles via present theory.

References:

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