# NONLINEAR MECHANICS OF WAVY INSTABILTY OF STEADY LONGITUDINAL VORTICES AND DRAG RISE IN BOUNDARY LAYER FLOW†

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*Summary* Wavy secondary instability of steady longitudinal vortices in boundary layer flow is studied. The nonlinear interaction problem is parabolized through scaling obtained from observations. Emphasis is placed on the nonlinear modification of the steady, averaged problem by the Reynolds stresses of the wavy disturbance. It is found that the skin friction in such a modification process increases well above the local turbulent boundary layer value.

## INTRODUCTION

Discussed in a recent paper [1] are the formulation and results for the nonlinear development of steady Taylor-Görtler vortices in the free mixing region after their release from upstream wall bounded flow These steady longitudinal vortices in turn, are susceptible to time-dependent wavy instabilities that are sometimes referred to as secondary instabilities [2],[3,[4]]. They are akinned to the unstable wavy vortices in a Taylor-Couette flow apparatus. At this Congress we will discuss the nonlinear mechanics of time-dependent instabilities of steady three-dimensional longitudinal vortices in the boundary layer. The steady mean flow could be Taylor-Görtler vortices in the boundary layer or artificially induced longitudinal vortices from spanwise-periodic roughness near the leading edge of the boundary layer. Emphasis is placed on the modification of the steady flow by the Reynolds stresses of the nonlinear wavy instabilities, which in turn, causes the skin friction drag to increase. The drag rise induced by the nonlinear wavy instability, a phenomenon which has escaped attention in previous nonlinear studies [5] is of an enormous important consequence in terms of momentum and scalar transport enhancement possibilities

## FORMULATION AND METHODOLOGY

It is much more physically illuminating to carry out the analysis in physical rather than in spectral coordinates. The flow is split into a time-independent mean flow and the time-dependent 'high frequency' instabilities; these are nonlinearly coupled via the Reynolds stresses of the unsteady component of flow. The total steady flow consists of longitudinal vortices and the basic steady boundary layer flow. The mechanics of nonlinear interactions between the steady flow component and the wavy instability are recoverable via time and spanwise averaging. Through scaling arguments, the steady flow component continues to be parabolic via scaling in its streamwise development in the viscous sense and devoid of curvature effects, except for the Görtler mechanism, under the assumption of thin boundary layers relative to the radius of curvature. The nonlinear wavy disturbance equations in physical variables are obtained by subtracting the Reynolds-averaged steady flow equations from the total flow quantities before the Reynolds splitting. The nonlinear disturbance equations are thus similar in form as in nonlinear hydrodynamic stability studies (such as in the general formulation in [5], [6]). Under the assumption of small boundary layer thickness relative to local radius of curvature of the wall, curvature effects in advective and viscous diffusion effects are negligible; the Görtler centrifugal mechanism in this case do not arise in the wavy disturbance problem as was found in the linear problem ([7], [2], [4]). Scaling arguments extracted from experiments [8] show that the time-dependent wavy instability velocities, unlike that of the steady flow, are very nearly isotropic; their physical wavy characteristics scale isotropically according to the local shear layer dimensions while their three-dimensional streamwise developing wave envelope would be parabolic, consistent with the steady mean flow. A number of simplifications are thus possible, leading to simpler understanding of the partial differential equations for the disturbances in the present case. The mean flow advection of disturbance momentum is attributed only to the streamwise mean flow velocity. Two dominant mechanisms for instability arises: one is the normal (or vertical) disturbance velocity advection of the three-dimensional structure of spanwise mean flow vorticity, a mechanism familiar in two-dimensional shear flow instabilities [9], the other is the spanwise disturbance velocity advection of the three-dimensional normal or vertical mean flow vorticity. These mechanisms were well identified in earlier linear studies [2], [3] and they remain so with nonlinearities taken into account. The nonlinear generation of higher harmonics is due to the time-dependent excess-Reynolds stresses familiar in nonlinear hydrodynamic stability studies [6].

The system consists of the downstream marching solution of the three-dimensional longitudinal vortex mean flow and of the coupled parabolic partial differential equations for the three-dimensional spectral components of the wavy disturbances, starting from imposed upstream initial conditions. Although the details are very much different, the physical implications of previous work on nonlinear hydrodynamic stability (e.g., [6]) are inferable from the present strongly nonlinear studies; which include the modification of the mean flow, the generation of higher harmonics and the effect of these on modifications of the fundamental component. These effects are distilled from the partial differential in the present case, reflecting the basic mechanisms depicted by the Stuart constants $k_{1,}$ $k_{2}$, and $k_{3}$ in the Stuart-Landau amplitude equation [6] which was rationally derived from the Navier-Stokes equations under the assumption of weak nonlinearity for parallel flow instabilities.

## RESULTS AND CONCLUDING REMARKS

Computational results are obtained for conditions corresponding to experiments [8] for the most amplified steady longitudinal vortex mode and the fundamental wavy instability mode at initial amplitudes consistent with observations. The mushroom shapes of the total steady flow streamwise velocity is shown to be modified as the wavy time-dependent instability amplifies into its nonlinear region. In the absence of wavy disturbance the steady flow skin friction can nearly bridge the transition from laminar values to that of turbulent, which is also the case with experimental measurements [8]. The effect of the wavy instability on the steady values of the skin friction is to increase it well beyond the local turbulent values as the flow develops downstream. This is shown in the accompanying figure.

Although the wavy instability momentum problem does not have the similarity properties with scalar transport as in the steady case [10] because of the streamwise pressure gradient in the wavy instability problem, nevertheless, one can approximately infer that the surface heat (and mass) transfer rates under longitudinal vortices and their wavy instabilities would be similarly augmented as would be found in measurements in progress at the Laboratoire de Thermocinétique in Nantes.

†In memoriam: Henryk Zorski

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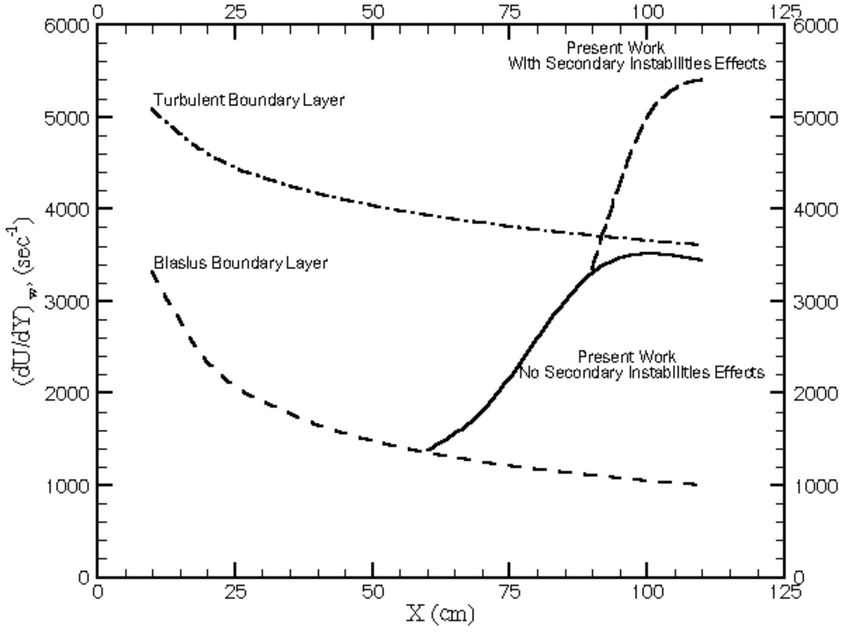


Shear Stress at the Wall