STABILITY OF FLOW IN A ROUGH CHANNEL

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Summary Theoretical and experimental investigation of stability of flow in a rough channel has been carried out. The flow destabilization predicted by the theory agrees with the destabilization observed in controlled experiments.

INTRODUCTION

Understanding how surface roughness affects the laminar-turbulent transition process is one of fundamental questions in fluid dynamics. This question has been posed by in 1883 [1] but, in spite of its long history, its rational resolution is still lacking. While this problem is of interest from the fundamental point of view, it is also of considerable practical importance in several application areas, e.g., design of large Reynolds number laminar airfoils, small Reynolds number turbulent airfoils, compact heat exchangers, laminar electrostatic precipitators, etc. The existing experimental investigations show that surface roughness contributes directly to the dynamics of flow only if the wall is hydraulically rough. Precise definition of hydraulical smoothness is not available due to the complex, flow-condition-dependent interaction between the roughness geometry and the moving fluid. The phenomenological effects of the "equivalent roughness" are summarized in the form of friction coefficient [2]. These concepts are being continuously re-evaluated [3,4].

THEORY

This work is focused on the analysis of stability of flow in a rough channel. The roughness geometry is represented in terms of Fourier expansion [5]. The steady flow in the rough channel is determined by solving the Navier-Stokes equations with spectral accuracy. The boundary conditions at the rough wall are enforced using (i) mapping technique [6] and (ii) immersed boundary conditions concept [7]. Both approaches produce the same results. The stability analysis is generalized to account for the spatial modulation of the flow induced by the roughness and is able to deal with disturbances whose structure is related to the roughness structure (the ratio of the respective wave numbers is rational) as well as with disturbances whose structure is not related to the roughness (the ratio of the wave numbers is irrational). Explicit results are given for two case studies, i.e., roughness represented by a single Fourier mode (wavy wall model) and roughness in the form of spanwise grooves (grooved wall model). Figure 1 displays neutral curves obtained for different values of the flow Reynolds number in the case of wavy wall model. It can be seen that the flow becomes unstable at $Re \approx 5000$ at this roughness amplitude and that the critical roughness wave number $\alpha \approx 10$. Results based on the grooved wall model (not shown) demonstrate that good approximation of the critical Reynolds number can be obtained using only the leading Fourier mode to represent roughness geometry.

EXPERIMENT

Experiments have been carried out in order to confirm theoretical predictions. The wind tunnel used for the measurements is able to produce supercritical laminar Poiseulle flow for Re up to Re=6000 in the case of smooth walls. Controlled disturbances are introduces and their evolution is measured in the streamwise direction. Performance of the facility has been verified by measuring evolution of Tollmien-Schlichting waves in the case of smooth walls. Subsequent measurement have been carried out in the case of corrugated wall with the corrugation described by a single Fourier mode with the corrugation wave number $\alpha=1.02$ and the corrugation amplitude S=0.04. Figure 2 displays results of measured growth rates for Re=4000 and Re=5000. It can be seen that the flow is unstable even at Re=4000 in the presence of surface corrugation. The range of unstable disturbance wave numbers is found to be somewhat larger then in the case of smooth channel. Figure 2 also displays predicted growth rates; good agreement between the theory and the experimental measurements is observed.

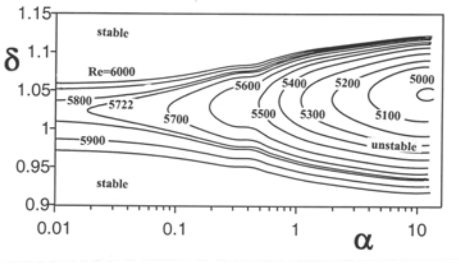

Figure 1 Neutral curves for the wavy wall model with roughness amplitude S=0.017 at different values of the flow Reynolds number Re as a function of the roughness wave number α and the disturbance wave number δ_r. Only two-dimensional disturbances are considered.

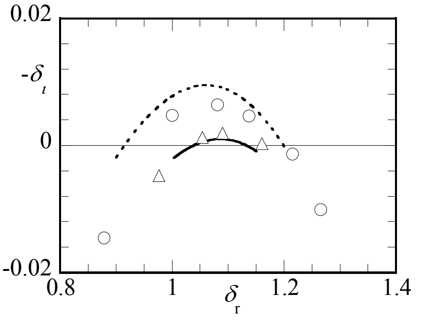

Figure 2. Variations of the amplification rate δ_i of disturbances in a Poiseuille flow at Re=4000 (triangles) and Re=5000 (circles) as a function of the disturbance wave number δ_r. One of the walls is smooth and the other has wavy form with the roughness wave number α=1.02 and the amplitude S=0.04.

SUMMARY

Theoretical and experimental investigation of stability of flow in a channel with distributed surface roughness has been carried out. Fourier expansions were used to model roughness geometry. Explicit results obtained in the case of wavy wall model (single Fourier mode roughness geometry) and grooved wall model (multi-Fourier modes roughness geometry) show that good approximation of the critical Reynolds can be obtained using leading Fourier mode to represent roughness geometry. Experiments carried out in the case of wavy wall model confirm theoretical predictions.

References:

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