COUPLED FREQUENCIES OF A FLUID-STRUCTURE INTERACTION CYLINDRICAL SYSTEM

Elena Gavrilova

Department of Mathematics, St. Ivan Rilski University of Mining & Geology, Sofia 1700, Bulgaria

Summary An upright fixed circular cylindrical tank with rigid bottom and side wall, where as a part of the side wall is an axial-symmetrical vibrating thin shell with clamped edges, is partly filled with an incompressible and inviscid fluid. Using the Bubnov-Galerkin method, the analytical solution of the problem about the determination of the free coupled vibrations of the obtained fluid-structure interaction system is found and some numerical examples are discussed.

INTRODUCTION

The study of the dynamic behavior of a tank and internal fluid is becoming of increasing importance in many fields such as the earthquake-proof design and the reliability assessment of fluid storage tanks. The problem about the determination of the coupled vibrations of systems fluid-tank with some elastic elements is under consideration by many authors. Bauer and co-authors [1] investigate the coupled oscillations of a cylindrical container with elastic walls and rigid bottom partially filled with liquid using the bending theory for the analysis of the container. The general problem of hydroelastic sloshing of fluid in a partially filled circular cylindrical container with rigid bottom and elastic walls is investigated in [2]. In [3] the liquid-filled flexible cylindrical container is considered as the velocity potential of a compressible fluid is found by the Galerkin method.

FORMULATION OF THE PROBLEM

An upright fixed circular cylindrical tank of diameter $2R$ and height L is partially filled with a homogeneous, incompressible and inviscid fluid to the depth H $(H < L)$.

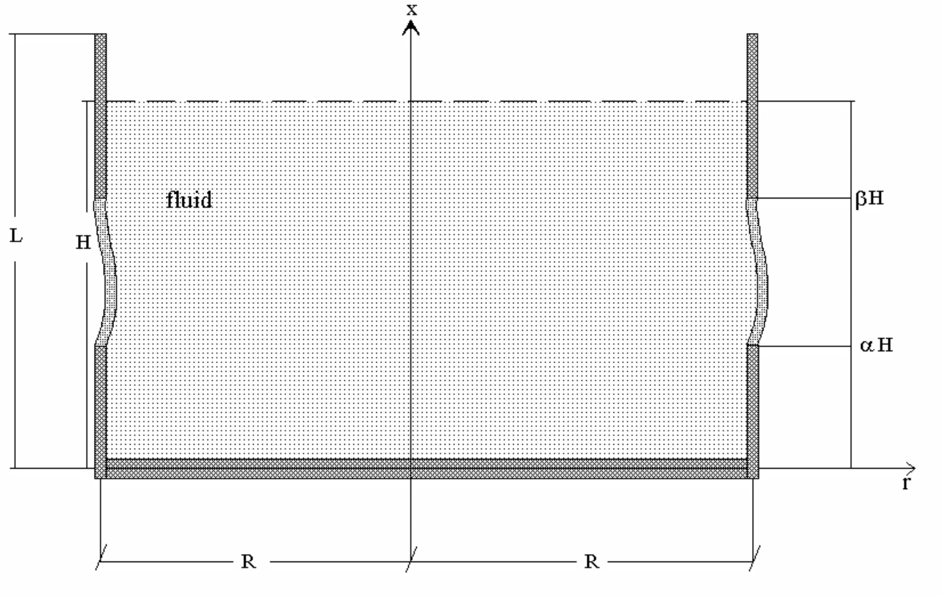

Figure 1. The geometry and co-ordinate system

The tank bottom at $x = 0$ (x is the vertical co-ordinate) and its side wall at $r = R$ are rigid as a part of the last one - $\alpha H \le x \le \beta H$ $(0 \le \alpha < \beta < 1)$ - is an axial-symmetrical vibrating thin shell with clamped edges, completely under the fluid. Its radial displacement along the outward normal is described with the function $w(x;t)$, where t is time. Such a system constitutes a fluid-structure interaction system. The fluid motion in the tank is supposed to be axisymmetrical and potential. Its velocity potential function $\varphi(r,x;t)$ is a solution of the partial differential equation

$$\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}+\frac{\partial^2\varphi}{\partial x^2}=0 \qquad 0 \le x \le H,\ 0 \le r \le R \tag{1}$$

with the boundary conditions:

$$\frac{\partial\varphi}{\partial x}=0 \quad \text{at the tank bottom } x=0,\ 0\le r\le R; \qquad \frac{\partial^2\varphi}{\partial t^2}+g\frac{\partial\varphi}{\partial x}=0 \quad \text{at the free surface } x=H,\ 0\le r\le R, \tag{2}$$

where g is the gravitational acceleration. The function $w(x;t)$ satisfies the partial differential equation

$$\frac{\partial^4 w}{\partial x^4}+\frac{Eh}{DR^2}w+\frac{\rho h}{D}\frac{\partial^2 w}{\partial t^2}=-\frac{\rho_0}{D}\frac{\partial\varphi}{\partial t}\Big|_{r=R} \qquad \alpha H\le x\le\beta H \tag{3}$$

$$w\Big|_{x=\alpha H}=w\Big|_{x=\beta H}=\frac{\partial w}{\partial x}\Big|_{x=\alpha H}=\frac{\partial w}{\partial x}\Big|_{x=\beta H}=0, \tag{4}$$

which express that the elastic shell is clamped along its edges. Here ρ_0 is the fluid density, h - the shell thickness, E - Young's modulus of elasticity of the shell material, ρ - the density of the shell material and D - its flexural rigidity. Equation (3) is derived based on the assumption that the longitudinal inertia of the elastic shell has negligible effect on its motion. Both functions $\varphi(r,x;t)$ and $w(x;t)$ must satisfy the compatibility condition

$$\frac{\partial\varphi}{\partial r}\Big|_{r=R}=\begin{cases}\dfrac{\partial w}{\partial t} & \alpha H\le x\le\beta H,\\ 0 & 0\le x\le\alpha H,\quad \beta H< x\le H.\end{cases} \tag{5}$$

ANALYTICAL SOLUTION

It is assumed that

$$\varphi(r,x;t)=i\omega\Phi(r,x)e^{i\omega t},\ w(x;t)=W(x)e^{i\omega t}, \tag{6}$$

where ω is the undetermined coupled natural frequency of the fluid-tank vibrations and $i^2=-1$. Using the equations (1)-(4), the following expressions can be written:

$$\Phi(x,r)=\sum_{n=1}^{\infty}A_n\cos(k_n x)I_0(k_n r),$$

$$W(x)=\frac{\rho_0\omega^2}{D}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}A_n\frac{I_0(k_nR)}{k_n^4+4\Omega^4}\sin\frac{m\pi(x-\alpha H)}{H(\beta-\alpha)}\left(a_{nm}+F_{1n}b_m+F_{2n}c_m+F_{3n}d_m+F_{4n}e_m\right), \tag{7}$$

where

$$\mathrm{tg}(k_nH)+\frac{\omega^2}{gk_n}=0,\qquad \Omega^4=\frac{1}{4D}\left(\frac{Eh}{R^2}-\rho h\omega^2\right), \tag{8}$$

and the constants a_{nm}, F_{in} $(n,m=1,2,3,\ldots;i=1,2,3,4)$ have completely determined values. After satisfying the compatibility condition (5) some infinite system of homogeneous algebraic equations about the unknown coefficients A_n is obtained. It will have a nontrivial solution, if the determinant of the coefficient matrix is equal to zero - thus the frequency equation is obtained.

The made numerical calculations show that the elastic part of the tank side wall lowers the natural frequency in comparison with the case of the rigid tank as the coupled frequency increases with the increase of the shell thickness and decreases with the increase of the fluid depth.

CONCLUSIONS

In this paper an upright fixed circular cylindrical tank with rigid bottom and side wall, where as a part of the side wall is an axial-symmetrical vibrating thin shell with clamped edges, is partly filled with an incompressible and inviscid fluid. The fluid motion in the tank is supposed to be axisymmetrical and potential.

Using the Bubnov-Galerkin method, the analytical solution of the problem about the determination of the free coupling vibrations of the obtained fluid-structure interaction system is found. Numerical examples show that the presence of some elastic shell in the tank side wall lowers the frequencies appearing.

The case with more elastic inclusions in the side wall of the tank can be considered in a similar way.

References

[1] Bauer H. F., Hsu T. -M., Wang J. T. -S.: Interaction of a Sloshing Liquid with Elastic Containers. *Trans ASME, J. Basic Eng.* **90**, Ser. D, No. 3: 373-377, 1968.

[2] Bauer H. F., Siekmann J.: Note on Linear Hydroelastic Sloshing. *ZAMM* **49**, Heft 10: 577-589, 1969.

[3] Fischer D.: Dynamic Fluid Effects in Liquid-Filled Flexible Cylindrical Tanks. *Earthq Eng. Str Dyn* **7**: 587-601, 1978.