LONG-WAVE MARANGONI INSTABILITY IN BINARY-LIQUID FILMS WITH SORET EFFECT

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Summary We study the onset of Marangoni instability of the quiescent equilibrium in a binary liquid film in the presence of Soret effect. Linear stability analysis shows that both monotonic and oscillatory long-wave instabilities are possible depending on the value of the Soret number χ . Sets of long-wave nonlinear evolution equations are derived for both types of instability. Bifurcation analyses are carried out for both types of instability.

INTRODUCTION

Linear and nonlinear analyses and investigation of pattern formation in long-wave Marangoni instability in a binary liquid layer open to the ambient gas phase are carried out, when the Soret effect is taken into account. The liquid layer is assumed to be sufficiently thin and with sufficiently high surface tension, so that the mathematical model that neglects both buoyancy-driven convection and deformation of the liquid-gas interface is valid

LINEAR STABILITY ANALYSIS

Linear stability analysis in the case of poorly conducting boundaries reveals that both monotonic and oscillatory modes of instability exist here. It is found that monotonic instability sets in when the Soret number χ exceeds the value $\chi_0 = -(1+L^{-1}+L^{-2})^{-1}$, where L^{-1} being the Lewis number, and the critical value of the Marangoni number is $M_{0,mon} = 48[1+\chi(1+L^{-1})]^{-1}$. The oscillatory instability emerges when $-1 < \chi < \chi_0$, and the corresponding critical value of the Marangoni number is $M_{0,osc} = 48(1+L)(1+\chi)^{-1}$.

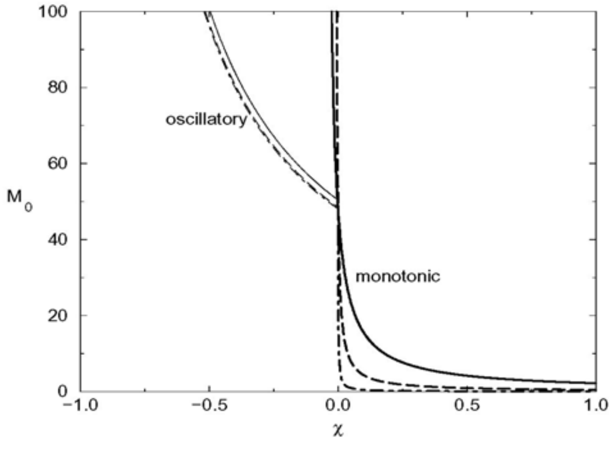

Figure 1. Neutral curves for both long-wave monotonic and oscillatory instabilities and for various values of the inverse Lewis number L^{-1} . The thick and thin curves correspond to the monotonic and oscillatory modes, respectively. The solid, long-dashed and dot-dashed lines correspond to $L^{-1} = 20, 100$ and 1000, respectively.

Figure 1 summarizes the results of linear stability analysis. It displays the neutral curves for both long-wave monotonic and oscillatory instabilities for $L^{-1} = 20, 100$ and 1000 . If the value of L^{-1} is fixed long-wave monotonic instability sets in for $\chi > \chi_0$. In the particular case of a pure liquid corresponding to $\chi = 0$ the instability is known to be monotonic with the critical Marangoni number $M_0 = 48$. At $\chi = \chi_0$ the long-wave oscillatory branch bifurcates off the monotonic one and manifests the instability threshold when $-1 < \chi < \chi_0$. This long-wave oscillatory instability disappears when $\chi \le -1$. The bifurcation structure remains the same for any $L^{-1} >> 1$. The critical Marangoni number for the oscillatory mode decreases with L^{-1} reaching a limiting curve $M_0 = 48(1+\chi)^{-1}$ in the limit of $L^{-1} \to \infty$. On the other hand, the critical Marangoni number for the monotonic mode increases with L^{-1} when $\chi < 0$, and decreases when $\chi > 0$.

NONLINEAR THEORY

Weakly nonlinear analysis of both monotonic and oscillatory instabilities yields sets of two nonlinear evolution equations that govern the spatio-temporal dynamics of the system. In the case of monotonic instability, as in the case of a pure liquid, one of the two equations is of evolution type, while the other is elliptic and describe convective effects in the layer that vanish when the Prandtl number of the liquid is large. Bifurcation analysis based on the evolution equation for a binary liquid with large Prandtl number yields amplitude equations for roll-, square- and hexagon patterns. It is shown that rolls patterns are unstable, while square patterns are stable in the physically relevant limit of large L^{-1}. Hexagonal patterns are found to bifurcate transcritically, and a steady stable hexagonal pattern in the system is possible. It is found that the hexagonal pattern appears in the subcritical region when the distance from criticality is small, and it is replaced by the square pattern at a finite value of this distance. In the case of oscillatory instability the set of nonlinear equations consists of two equations of evolution type. It is found that bifurcation of both standing and traveling waves is supercritical in the range $\chi_1 < \chi < \chi_0$, where $\chi_1 = 3L + O(L^2)$. In this range of the Soret number standing waves are found to be unstable, while travelling waves appear to be stable.

It is found that if finite deformations of the interface are allowed binary-liquid films rupture in a finite time in both settings of prescribed temperature distribution and temperature gradient at the solid substrate. The behavior of the film near rupture is investigated.

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