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POROUS ANISOTROPIC COMPOSITES UNDER MICROFRACTURE

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Summary The aim of the present paper is to study the behavior of the matrix composite materials of stochastic structure weakened by randomly distributed microcraks. It is considered the case of transversally-isotropic components. It is assumed that the matrix is porous and the loading processes leads to the microdestruction and the accumulation of damage in it. As basic relations the stochastic equations of elasticity theory and porosity balance equation are taken.

MECHANICAL MODEL

Let's consider a representative volume of composite of stochastic structure with transversally-isotropic components and porous matrix at given macrodeformations and suppose, that the pores shape is quasispherical. The effective deformative properties and the strain-stress state of material are determined on the basis of stochastic equations of the elasticity theory, allowing for an accidental character of microdestructions, by a method of conditional moment functions. Then Hook's low will connect the macroscopic stresses and strains fields of such material by the following equation:

$$\langle \sigma_{ij} \rangle = \lambda^{*}_{ij\alpha\beta} \langle \varepsilon_{\alpha\beta} \rangle . \qquad (1)$$

Here $\lambda^{*}_{ij\alpha\beta}$ - tensor of effective elastic constants. The effective elastic constants are the functions of the elastic moduli tensors of composite components $\lambda^{[1]}_{ij\alpha\beta}$ and $\lambda^{[2]}_{ij\alpha\beta}$, their volume concentrations c_1, c_2, porosity p_2 of matrix and parameter t characterizing the shape of inclusions. Indexes *1* and *2* denote inclusions and matrix respectively:

$$\lambda^{*}_{ij\alpha\beta} = \lambda^{*}_{ij\alpha\beta}\left(\lambda^{[1]}_{ij\alpha\beta}, \lambda^{[2]}_{ij\alpha\beta}, c_1, p_2, t\right). \qquad (2)$$

If macrodeformations of composite and effective elastic moduli tensor are known we can define the matrix stresses:

$$\langle \sigma_{ij} |_2 \rangle = \lambda^{[2]}_{ij\alpha\beta}\left(I_{\alpha\beta kl} + c_2 \left(\langle \lambda_{\alpha\beta kl} \rangle - \lambda^{*}_{\alpha\beta kl} \right) {\lambda^{[3]}_{kl\gamma\rho}}^{-1} \right) \langle \varepsilon_{\gamma\rho} \rangle \; ; \qquad (3)$$

here,

$$\langle \lambda_{\alpha\beta kl} \rangle = c_1 \lambda^{[1]}_{\alpha\beta kl} + c_2 \lambda^{[2]}_{\alpha\beta kl} \; ; \; \lambda^{[3]}_{kl\gamma\rho} = \lambda^{[1]}_{kl\gamma\rho} - \lambda^{[2]}_{kl\gamma\rho} . \qquad (4)$$

Stresses averaged over matrix skeleton are connected with stresses in porous matrix $\langle \sigma_{ij} |_2 \rangle$ by equations:

$$\langle \sigma^2_{ij} \rangle = \langle \sigma_{ij} |_2 \rangle \big/ (1 - p_2) . \qquad (5)$$

According to formulas (3) – (5) the stresses $\langle \sigma^2_{ij} \rangle$ averaged over the matrix skeleton are connected with macrodeformations as follows:

$$\langle \sigma^2_{\alpha\beta} \rangle = \lambda^{[2]}_{\alpha\beta mn}\left(I_{mnkl} + c_2 \left(\langle \lambda_{mnkl} \rangle - \lambda^{*}_{mnkl} \right) {\lambda^{[3]}_{kl\gamma\rho}}^{-1} \right) \langle \varepsilon_{\gamma\rho} \rangle \big/ (1 - p_2) . \qquad (6)$$

For an anisotropic material, the simplest structural model of microdamageability is constructed on the basis of the strength criterion for a macrovolume as the limiting value of the second invariant of the deviator of the average-stress tensor for the undamaged part of the material, i.e., the Huber-Mises criterion. In the considered case (a transversally isotropic material with isotropy plane $x_1 x_2$), we proceed from a generalization of the Huber-Mises criterion in the form:

$$\bar{J}^{2}_{\sigma} = \sqrt{\langle \sigma^2_{ij} \rangle' \langle \sigma^2_{ij} \rangle' + a_1 \langle \sigma^2_{33} \rangle'^2 + a_2 \left(\langle \sigma^2_{11} \rangle' + \langle \sigma^2_{22} \rangle' \right) \langle \sigma^2_{33} \rangle' + a_3 \left(\langle \sigma^2_{13} \rangle'^2 + \langle \sigma^2_{23} \rangle'^2 \right)} = k_2 , \qquad (7)$$

where parameters a_1, a_2, a_3 are dimensionless constants characterizing the transverse-isotropy of the strength properties of the matrix skeleton and k_2 is the limiting significance of the corresponding expression, which is a random function of coordinates. For $a_1 = a_2 = a_3 = 0$, the Huber-Mises criterion follows from (7).

The single-point distribution function $F\ (k)$ of parameter can be represented by the exponential-power distribution function in a semi-infinite domain, i.e., the Weibull distribution:

$$F(k_2) = \begin{cases} 0 & , k_2 < k_0 \\ 1 - \exp\left(-n (k_2 - k_0)^{\alpha} \right) & , k_2 \geq k_0 \end{cases} . \qquad (8)$$

Here k_0 is the lower limit value of the intensity of the average tangential stresses over matrix skeleton, from which the destruction begins, n and α - the parameters selected from the condition of the most approximation of strength spread. They are determined by experimentally for the concrete material.

If the stresses of matrix skeleton $\langle \sigma_{ij}^{2} \rangle$ are known, then formulas (7), (8) determine the relative content $F(k_2)$ of destroyed microvolumes in matrix. If the pores are modeled by the destroyed microvolumes, we can write the porosity balance equation:

$$p = p_2 + F(k_2)(1 - p_2)\ . \tag{9}$$

The matrix skeleton stresses $\langle \sigma_{ij}^{2} \rangle$ can be defined by the composite macrostrains $\langle \varepsilon_{\alpha\beta} \rangle$ in accordance with given above formulas (2) – (6). These relations allow to determine the dependence of the current porosity of matrix p, caused by microdestruction, at the given macrostrains. Then, substituting p instead of p_2 into equations (2) – (6), we can obtain the nonlinear dependence of the macrostresses on the macrostrains, caused by microdestruction. These relations take into account the strength spread of material.

The relations (2) – (9) at the given functions of distribution (8) represent the nonlinear equations for determination of porosity at given macrodeformations $\langle \varepsilon_{ij} \rangle$. Their solution can be constructed by iterative methods.

BIAXIAL EXTENSION OF TRANSVERSALLY-ISOTROPIC COMPOSITE

Using described above methods and obtained equations of a porosity balance, as an example, the nonlinear diagrams of macrodeformation are constructed and the behaviour of a transversally-isotropic porous material for biaxial expansion is researched:

$$\left(\langle \varepsilon_{11} \rangle \neq 0, \quad \langle \varepsilon_{22} \rangle = 0.002 \right),$$

for given elastic moduli of inclusions and matrix:

$$\lambda_{11}^{[1]} = 118.4\ GPa,\ \lambda_{33}^{[1]} = 107\ GPa,\ \lambda_{13}^{[1]} = 32\ GPa,\ \lambda_{12}^{[1]} = 19\ \ GPa,\ \lambda_{44}^{[1]} = 35\ \ GPa,$$

$$\lambda^{[2]} = 3.7\ \ GPa,\ \mu^{[2]} = 1.1\ \ GPa, \tag{19}$$

the volume concentration of inclusions and pores in matrix and the shape of inclusions:

$$c_1 = 0.4\ , \qquad p_2 = 0\,;0.2\,;0.4\ , \qquad t = 2\ , \tag{20}$$

without allowance for strength spread and at given parameters of the strength distribution function of matrix material:

$$\alpha = 2, \quad n = 10^{\ 3};\ 2 \times 10^{\ 3}\ , \quad k_0 = 0.015\ \ Gpa. \tag{21}$$

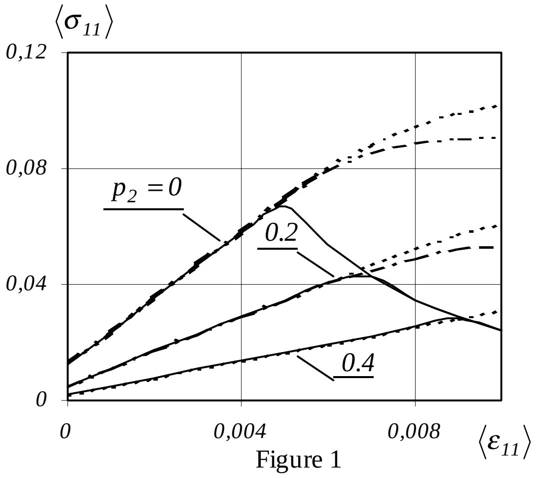

Figure 1

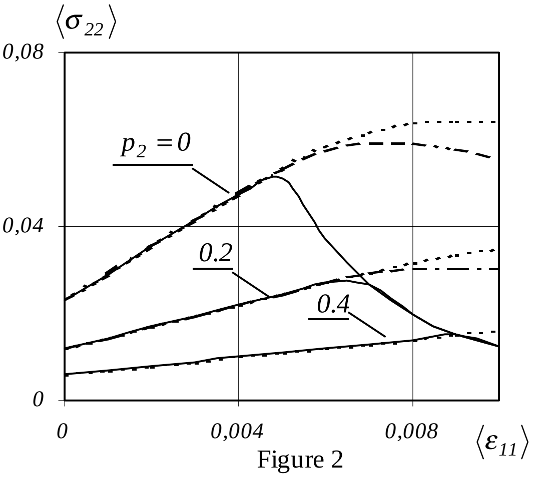

Figure 2

References

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