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PDF COMPUTATION OF TURBULENT FLOWS WITH A NEW NEAR-WALL MODEL

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Summary The modeling and computation of near-wall turbulent flows is addressed with the probability density function (PDF) method for velocity and the turbulent frequency. A new model for viscous transport is proposed and a method of elliptic relaxation for a blending function is applied to model the pressure-strain term. The PDF equation is solved by a Monte Carlo method and the whole approach appears as a self-contained Lagrangian simulation using stochastic particles. For the sake of numerical example, the fully developed channel flow is computed; results are compared with the available DNS data.

PDF modelling of the near-wall effects

From a practical point of view, a sound modeling of near-wall turbulence is of utmost importance, for the overall characteristics of momentum and heat transfer are mostly controlled and determined by a relatively thin near-wall region. At the same time the near-wall treatment in turbulent flows remains a notorious difficulty for statistical modeling. In the wall proximity a strong inhomogeneity due to the mean shear and considerable gradients of turbulence statistics are observed. The turbulence in this region is also highly anisotropic, with the wall-normal direction definitely distinct from others; this effect is felt also at larger distances from the wall through the pressure field. In the near-wall region, the molecular transport (viscosity, thermal conductivity) has to be accounted for explicitly.

The present work addresses the modeling and computation of turbulent wall-bounded flows. Approach followed here is the stand-alone PDF method, cf. [7], which is extended to account for wall effects. This is also the continuation of the previous work of the authors [5], [8], where the wall function approach was applied in the PDF method. The work differs in a few aspects from the study of Dreeben and Pope [1]. First, a new model for viscous transport is proposed, which operates with only first-order derivatives of the mean velocity field. In order to model the non-local wall effects we implement in the PDF approach the elliptic relaxation method [2] in its efficient one-equation variant [3]. In order to deal with numerical problems arising in the modeling of the viscous sublayer, a new integration scheme for the stochastic differential equations is proposed.

In the Lagrangian approach the viscous effects are modelled through the Brownian motion in the equation for the stochastic particle position and additional terms in equation for velocity increment [1]. The alternative model presented below retains the same structure of the velocity increment equation as the model for high Reynolds numbers [7]:

$$\begin{aligned} d\mathcal{X}_i &= \mathcal{U}_i dt + \sqrt{2\nu}\, dW_i^X, \\ d\mathcal{U}_i &= -\frac{\partial\langle P\rangle}{\partial x_i}\, dt - (A_{ij}+G_{ij})\,(\mathcal{U}_j - \langle U_j\rangle)\, dt - \frac{1}{2}\frac{\epsilon}{k}\,(\mathcal{U}_i - \langle U_i\rangle)\, dt + \sqrt{D}\, dW_i \end{aligned} \tag{1}$$

where $\langle\cdot\rangle$ denotes the ensemble average, P is the kinematic pressure, $\boldsymbol{\mathcal{U}}(t)$ is the velocity of a particle, defined as the Eulerian fluid velocity $\mathbf{U}(\mathbf{x}, t)$ evaluated at the particle position, i.e. $\boldsymbol{\mathcal{U}}(t) = \mathbf{U}\,[\boldsymbol{\mathcal{X}}(t), t]$; $d\mathbf{W}$ and $d\mathbf{W}^X$ denote the increments of the Wiener process, ϵ is the dissipation rate of the turbulent kinetic energy k, and $D = (2/3)G_{kl}\langle u_k u_l\rangle$. The tensor A_{ij} is determined from the requirement

$$A_{il}\langle u_l u_j\rangle = \nu \frac{\partial\langle U_i\rangle}{\partial x_k}\frac{\partial\langle U_j\rangle}{\partial x_k}. \tag{2}$$

The LHS of the above expression has the physical interpretation: it is the dissipation rate of the kinetic energy connected with the mean motion. The form (2) assures that in the Reynolds stress equations corresponding to the stochastic model (1) the viscous transport term is exact.

In the vicinity of the wall the effects of kinematic damping of the wall-normal velocity component and pressure scrambling can be modeled by a proper form of the tensor G_{ij}. Here, the components of this tensor are computed as the interpolation of their known near-wall form G_{ij}^w and a standard quasi-homogeneous model G_{ij}^h (eg. the basic pressure-strain model) used far from walls: $G_{ij} = (1-\alpha\epsilon T)G_{ij}^w + \alpha\epsilon T\ G_{ij}^h$, where $T = 1/\langle\omega\rangle$ is the time scale, $\langle\omega\rangle$ is the mean turbulent frequency and α is the elliptic blending function to be determined from the Helmholz equation [3]:

$$L^2\nabla^2\alpha - \alpha = -\frac{1}{\epsilon T} \tag{3}$$

with the length scale $L = C_L \max\{\frac{k^{3/2}}{\epsilon}, C_\eta \frac{\nu^{3/4}}{\epsilon^{1/4}}\}$. In the Lagrangian formulation the equation for the turbulent frequency ω is solved and ϵ is computed from the formula $\epsilon = k\langle\omega\rangle + \nu C_T^2\langle\omega\rangle^2$ (see [1]).

The dissipation rate ϵ has a non-zero value at the wall while the kinetic energy $k \sim y^2$ in the wall vicinity. Hence, some expressions present in the stochastic differential equations (1) are unbounded when discretized with the Euler scheme: $(\epsilon/k)u_i\Delta t \sim 1/y$ for $i \neq 2$. For the purpose a new numerical scheme has been developed [4]. It is based on the exponential solution of the stochastic equations (1) (details are not shown here).

PDF computation results for channel flow

Another objective of the work is to report computational results from the velocity PDF code for the case of fully developed channel flow. Computations are done to match the numerical experiment (DNS) [6] at $Re_\tau = 395$ and 590. The statistics (cf. Fig. 1) fit reasonably well the DNS data, although the mean velocity profile show a discrepancy in the buffer layer. This can be a consequence of the simple one-equation elliptic blending model used. Figure 2 presents probability density functions of the fluctuating streamwise velocity computed at different distances from the wall. As it is seen, even the definitely non-Gaussian behavior of the probability density functions near the wall can be reproduced correctly by the PDF method.

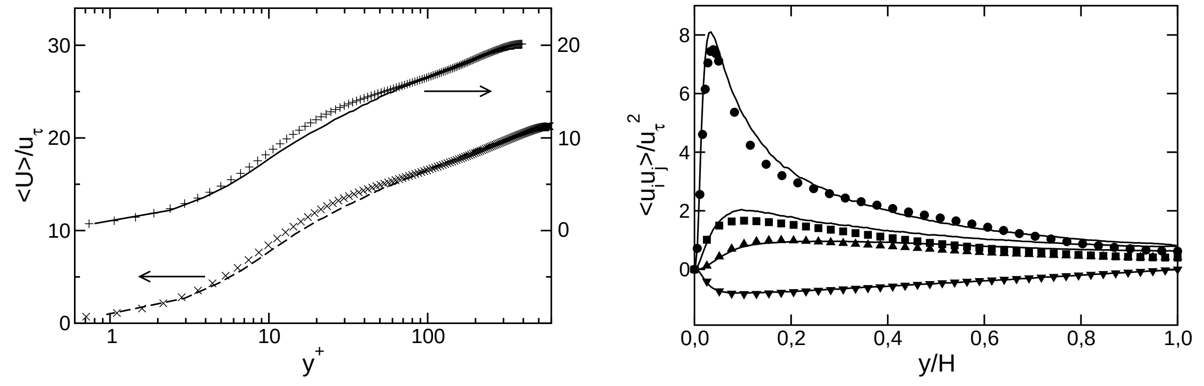

Figure 1. a) Mean velocity profiles in the turbulent channel flow at $Re_\tau = 590$ and $Re_\tau = 395$ (data sets shifted upward by 10); PDF computations: lines, DNS results[6]: symbols. b) the Reynolds stresses at $Re_\tau = 395$, PDF computations (_____); DNS data: $\langle u^2 \rangle$ ●, $\langle v^2 \rangle$ ▲, $\langle w^2 \rangle$ ■, $\langle uv \rangle$ ▼.

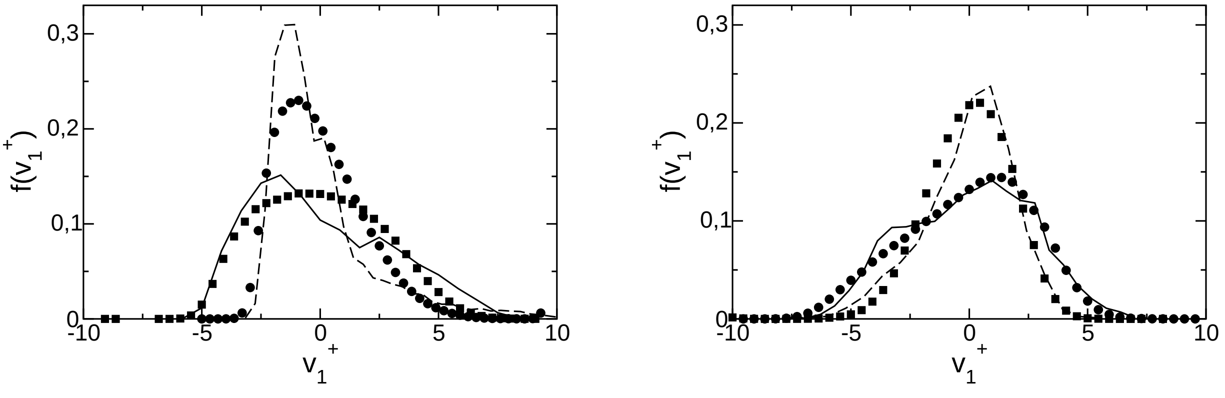

Figure 2. PDFs of the streamwise velocity fluctuations at $Re_\tau = 395$. a) PDF computations at $y^+ = 5$ (__ __) and $y^+ = 10$ (_____). b) PDF computations at $y^+ = 20$ (_____) and $y^+ = 80$ (__ __). DNS: symbols.

Conclusions

The main thrust of the work has been the development of a complete near-wall stochastic model with a new proposal for viscous transport. Unlike the previous model [1] no second order derivatives need to be computed. Moreover, the Lagrangian equation for velocity increment retains the same simple structure as for the high-Re case. The elliptic relaxation approach has been applied in the Lagrangian PDF setting in its simple and computationally efficient variant [3] with only one additional elliptic equation solved. When supplemented with a suitable scalar transport equation, the approach presented here can readily be applied to the case of near-wall turbulence with heat transfer to model the thermal fluctuations in the immediate vicinity of the wall.

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