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HYDRODYNAMIC INTERACTION OF A SPHERICAL PARTICLE IN POISEUILLE FLOW BETWEEN PLANAR WALLS

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Summary By using a two-dimensional Fourier representation of the Green tensor for Stokes flow between parallel walls, we calculate all friction and mobility functions for a single spherical particle moving between two walls subject to Poiseuille flow. The method readily generalises to N particles. For a channel narrow with respect to particle size, superposition of one-wall results is not an accurate approximation for the two-wall problem. Translation-rotation coupling is significant and changes its sign as the lateral position of the particle ranges across the channel. We illustrate these two-wall effects by calculating the trajectories of a magnetic colloid particle in Poiseuille flow subject to an external magnetic field.

INTRODUCTION

Hydrodynamic forces between particles play an important role in determining the dynamics of bulk suspensions of mesoscopic particles [1]. In addition to particle-particle forces, hydrodynamic interactions between particles and surrounding walls can play a significant role in sedimentation [2] and in the dynamics of bound particle clusters [3]. In recent years the development of optical tweezers and confocal microscopy has enabled one to measure directly some of the hydrodynamic interactions between one particle and a single wall [4] or between one particle and two parallel walls [5, 6] as well as hydrodynamic interactions between two particles in the presence of plane walls [7]. These experimental advances and the possibility of technological applications to biological sensors and catalysis based on mesoscopic particles flowing in suspension near walls means that there is need of a better understanding of particle-wall hydrodynamic interactions in channel flow.

TWO-WALL GREEN TENSOR

For a spherical particle near a single wall we have shown earlier that friction functions and mobilities may be calculated from the matrix elements with respect to a complete set of vector harmonics of the Green tensor for the Stokes flow equations in the presence of the wall [9]. Inversion and projection of this matrix gives the grand resistance matrix whose inverse defines the mobility matrix. The same formal scheme works also for two walls, however, the difficulty lies in obtaining a tractable form of the Green tensor. In earlier single wall studies we used an image technique [8, 9] to calculate the Green tensor. For two walls, the Green tensor has been obtained some time ago by a mixture of image methods and two-dimensional Fourier transformation [10]. More recently a scheme has been proposed which builds on the one wall image technique but involves infinite sums of images in the two walls [11]. By use of a two-dimensional Fourier representation without recourse to image summation we have obtained the exact Green tensor for Stokes flow between parallel walls in a representation which is much more symmetric than the earlier calculation [10].

ONE-PARTICLE FRICTION MATRIX

Our Fourier decomposition makes it possible to reduce to one-dimensional quadrature the calculation of all matrix elements of the Green tensor with respect to a complete set of vector harmonics. In obtaining this essentially exact result we derive exact formulae for a class of integrals which involve a scalar spherical harmonic weighted with a complex exponential function. By inversion of this Green tensor matrix followed by projection onto a finite dimensional space of low-order harmonics we obtain the grand resistance matrix for the spherical particle in the presence of the two walls. We determine the Cartesian form of the grand resistance matrix in the presence of an imposed Poiseuille flow between the two walls and relate the Cartesian representation to the harmonic representation. From the grand resistance matrix and its inverse we obtain all scalar friction and mobility functions for a single spherical particle between two walls and subject to an incident Poiseuille flow.

NUMERICAL CALCULATION

The numerical evaluation of the friction and mobility functions involves numerical integration followed by numerical matrix inversion. By comparison with the numerical boundary collocation calculations of Ganatos et al [12, 13] we show the rapid convergence of our scheme. We compare the exact two-wall result with the result for a single wall and with a superposition approximation. When the ratio of wall spacing to particle diameter is less than about five we find that simple superposition of single wall results is not an accurate approximation. The translation-rotation coupling changes sign as the lateral position of the particle ranges between the walls. We show the scalar translation-rotation friction function $\psi^{\rm tr}(R_z/a)$ below for a ratio of wall spacing to particle diameter equal to two. Here the two walls are located at $R_z = \pm W$ where R_z is the lateral distance of the sphere centre from the centre of the channel and a is the sphere radius.

APPLICATIONS AND CONCLUSIONS

From the friction and mobility functions we can carry out Stokesian dynamics studies of the trajectory of a particle in Poiseuille flow and subject to external fields. In particular, for polar particles subject to external electric or magnetic fields the translation-rotation coupling converts external torque into translational motion. We have studied this effect for a range of field strengths and field orientations. The single particle results are quite interesting in their own right but an important advantage of our method is that it can be generalized quite simply to N particles rather than to just one. For the many-particle problem we require matrix elements of the Green tensor between two centres rather than the same centre. The Fourier representation allows us again to reduce the calculation of all matrix elements to quadrature thus opening the way to N-body Stokesian dynamics studies of a suspension between two walls.

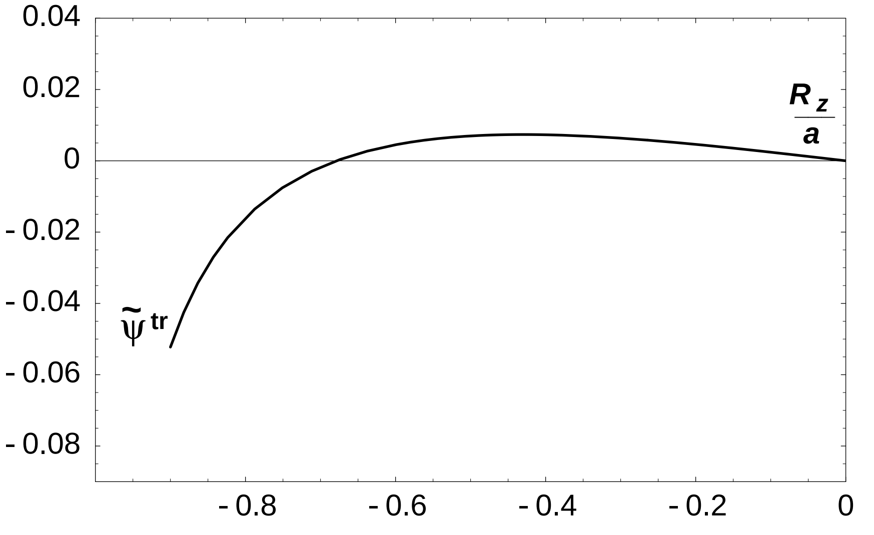

Figure 1. ψ^{tr} as a function of R_z/a for $W = 2a$ and $-0.9 \leq R_z/a \leq 0$.

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