XXI ICTAM, 15-21 August 2004, Warsaw, Poland

ID 10580

METHOD FOR SOLVING NONLINEAR PROBLEMS ON UNSTEADY FREE-BOUNDARY FLOWS

Yuriy A. Semenov,
Institute of Technical Mechanics of the NAS and NSA of Ukraine, 15 Leshko-Popel' Street, Dniepropetrovsk, 49005, Ukraine, e-mail: relcom@semenov.dp.ua

Summary *A direct method of finding a flow potential of 2-D inverse boundary-value problems is proposed. The method potentialities are illustrated by solving various new water impact problems.*

Historically, a progress in solving the problems of free boundary potential flows in an exact mathematical formulation is based on the development of the complex variable functions theory. Since any analytical function meets the requirements of a fluid incompressibility and zero vorticity the problem is to find such analytical function, which satisfy to given boundary conditions. The problems of free boundary flows lead to mixed boundary conditions. A velocity modulus on the free boundary is known from Bernoulli/Cauchy–Lagrange integral, and a body shape determines the velocity direction. A conformal mapping method proposed by Helmholtz and Kirchhoff and methods developed by Zhykovsky, Levi-Chivita, Chaplygin are applicable for solving the problems on steady free boundary flows. Nonlinear problems of unsteady free flows remain intricate problems. Only few peculiar examples are presented in literature. The first is a problem of an unsteady cavity flow past a flat plate solved by Karman (1949) under condition of a fixed free boundary in time. The second is a self similar problem of water entry of a wedge solved by Dobrovol'skaya (1969).

We propose a way of determination of a complex function on its modulus and argument or its real part and argument, which are given on the boundary of a simply connected domain. In combination with Chaplygin's singular point method it makes possible to determine the expressions of a complex velocity and a derivative of the complex potential of an arbitrary unsteady free boundary flow. These expressions contain in explicit form the modulus and the argument of the velocity as functions of a parameter variable and time. The dynamic and kinematic boundary conditions lead to a system of the integral and integro-differential equations for determination of these unknown functions.

At the first stage we apply this method for solving various water impact problems. This kind of problems contains such important features as nonlinearity, unsteadiness and three phase contact points.

Oblique water entry of a wedge. The self-similar solution that characterizes the flow about a wedge entering a liquid surface, originally at rest, is here considered as a reverse flow in a frame of reference attached to the impacting body. Let *x, y* be Cartesian co-ordinates with their origin ***O*** located at the contact point of the right wedge side with the free surface. The half-space of the ideal weightless incompressible fluid has an inflow velocity v_∞ and an angle γ with the *y*-axis. The sketches of the fluid domain and of the parameter space are shown in Figure *1a* and *1b*, respectively. The deadrise angles on the right and left hand sides are $\beta_R = \pi/2 - \alpha + \delta$ and $\beta_L = \pi/2 - \alpha - \delta$, where 2α is the wedge angle and δ is the deviation angle, that is the angle between the *y*-axis and symmetry axis of the wedge. In the case of a constant entry velocity the self-similar variables $\tilde{x} = x/(v_0 t)$, $\tilde{y} = y/(v_0 t)$ can be introduced so that the time dependent fluid domain in the physical plane is mapped into a stationary domain in the $\tilde{x}, \tilde{y}$ plane and the complex velocity potential $W(z,t) = \phi(z,t) + i\psi(z,t)$ takes the form

$$W(z,t) = \phi(z,t) + i\psi(z,t) = v_0^2 t\left[\tilde{\phi}(\tilde{z}) + i\tilde{\psi}(\tilde{z})\right] \quad (1)$$

where $\tilde{\phi}$ are the velocity potential and $\tilde{\psi}$ the stream function in the stationary plane $\tilde{z} = \tilde{x} + i\tilde{y}$.

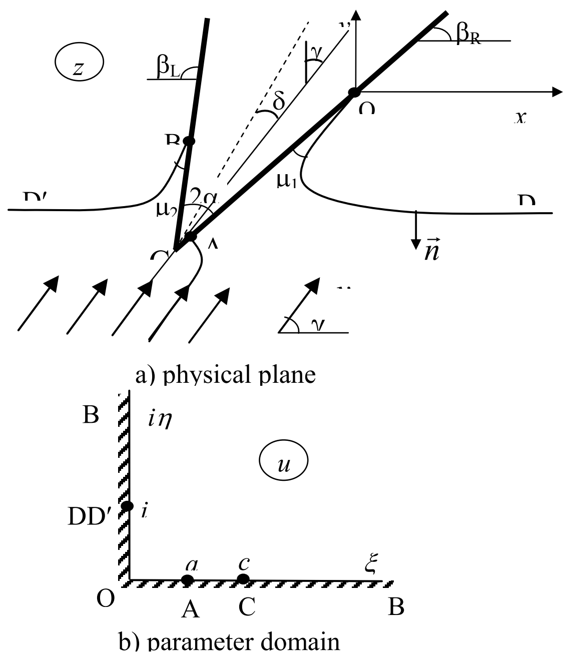

Fig.1. Inflow of a half-plane of the liquid on an inclined wedge

The expression of the complex velocity potential $\tilde{W}$ can be found by constructing the expression of the complex velocity and derivation of the complex potential in the parameter domain. Using the proposed method the final expression of the complex velocity takes the form

$$\frac{d\tilde{W}}{d\tilde{z}} = e^{-i(\gamma_\infty + \alpha + \delta)}\left(\frac{u-a}{u+a}\right)\left(\frac{u+c}{u-c}\right)^{(1-2\alpha/\pi)} \times \exp\left[-\frac{i}{\pi}\int_0^\infty \frac{d\ln v}{d\eta}\ln\left(\frac{i\eta - u}{i\eta + u}\right)d\eta\right] \quad (2)$$

Setting $u = \xi$ in equation (2), along the wetted part of the wedge it is $\arg\left(d\tilde{W}/d\tilde{z}\right) = -\beta_R$ for $0 < \xi < c$, ID 10580

and $\arg\left(d\widetilde{W}/d\widetilde{z}\right) = -(\pi - \beta_L)$ for $c < \xi < \infty$. On the free surface ($u = i\eta$) the modulus of equation (2) equals the function $v(\eta)$ which is determined later on by using the dynamic boundary condition.

Analyzing the singularities of the complex potential we can construct the expression of derivative of the complex potential

$$\frac{d\widetilde{W}}{du} = N u^{(2\mu_1/\pi - 1)} \frac{u^2 - a^2}{\left(1+u^2\right)^2} \exp\left[-\frac{1}{\pi}\int_0^\infty \frac{d\theta}{d\eta}\ln\left(\eta^2 + u^2\right)d\eta\right]. \quad (3)$$

where N is the scale factor, and the function $\theta(\eta) = \arctan\left(v_s / v_n\right)$ is expressed via the tangential $v_s(\eta)$ and normal $v_n(\eta)$ velocity components on the free boundary. From equation (3) it follows that, on the wedge sides ($u = \xi$), the imaginary part of the complex potential is equal to zero while along the free boundary

($u = i\eta$) it holds $\dfrac{\operatorname{Im} d\widetilde{W}}{\operatorname{Re} d\widetilde{W}} = \dfrac{v_n}{v_s}$. Thus, equation (3) satisfies the definition of the complex potential.

The following conditions: the inflow velocity, the length of the wetted part of the wedge side, the y-coordinate at the infinity at the right equals one at the left make it possible to determine the parameters a, c, N. The unknown functions $v(\eta)$ and $\theta(\eta)$ is determining form the dynamic and kinematic boundary conditions.

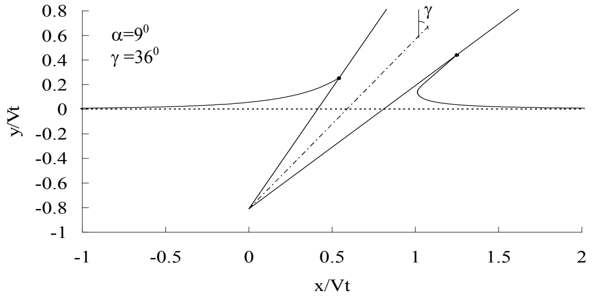

Fig.2 Solution for oblique entry of the wedge case with $\alpha = 9^0$ $\gamma = 36^0$

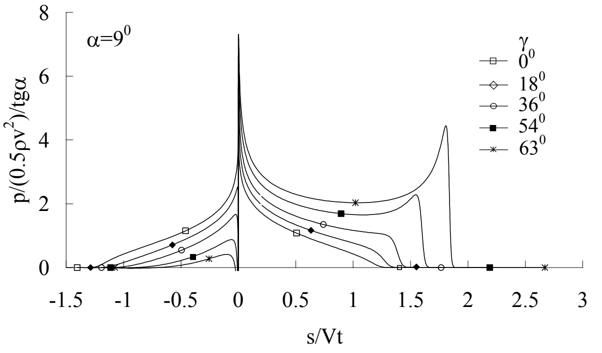

Fig.3 Pressure distribution on the right side (s>0) and on the left side (s<0) of the wedge

Oblique entry of a flat plate. This self-similar problem also was solved by the same way. Analyzing the flow singularities the expression of the complex velocity and the derivative of the complex potential were constructed. These expressions are similar to the Eq.(1) and (2). In Fig. 3 and Fig.4 are shown the free surface shape and pressure distribution along the flat plate. The black line in Fig.4 corresponds to the D.P.Wang's linear theory. When the angle of attack tends to zero the obtained results and the results of the linear theory are very closely.

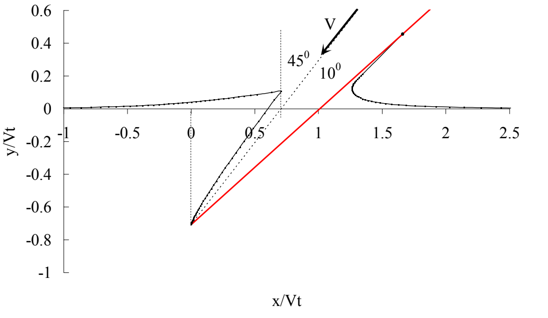

Fig.3 Solution for oblique entry of the flat plate case of the angle of attack $\alpha = 10^0$ and the velocity inclination $\gamma = 45^0$

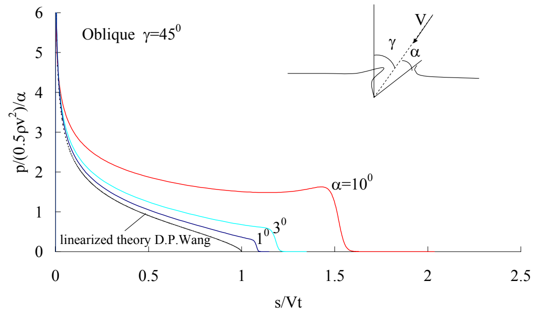

Fig.4. Pressure distribution on the flat plate

Liquid wedge impacting a solid wall. The self-similar problem was solved by the same way. This is a first obtained complete solution of the nonlinear problem.

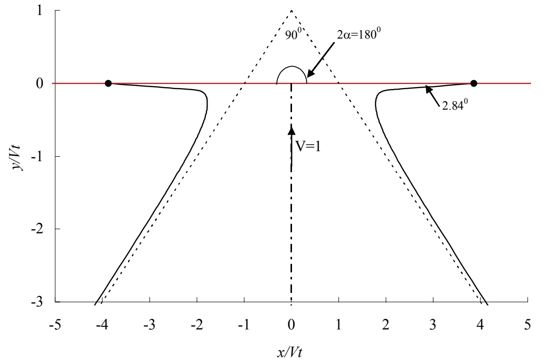

Fig.5. Free boundary of a liquid wedge impacting the solid wall

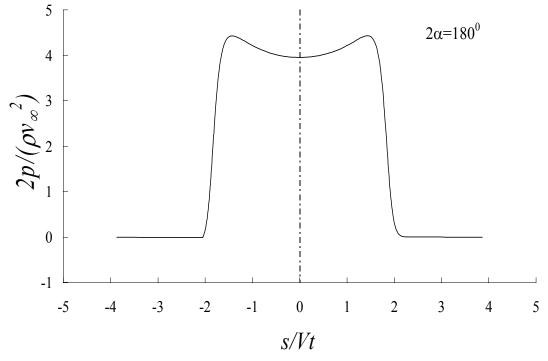

Fig.6. Pressure distribution along the wall