HOMOCLINIC ORBITS LAYERING IN THE COUPLED ROTOR NONLINEAR DYNAMICS AND CHAOTIC CLOCK MODELS: A PARADIGM FOR VIBRATIONS AND NOISE IN MACHINES

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Summary: For examining natural clocks of reductor (power transmission), as well as source of nonlinear vibrations and noise in its dynamics, it is necessary to investigate properties of nonlinear dynamics, and phase portraits, as well as structures of homoclinic orbits, layering and sensitivity of this layering of homoclinic orbits and bifurcation of homoclinic points. In the paper, the natural clocks of nonlinear dynamics of coupled rotors are studied.

Key words: *Homoclinic orbits and points, separatrice layering, trigger, coupled singularities, bifurcation, vector method, mass moment vectors, phase plane and portrait, couple triggers, the form of number eight or it multiplicand, Chaotic Clock Models.*

INTRODUCTION

Dynamics of coupled rotors (see Fig. 1) and of gyrorotors are very old engineering problems with many different search results and discoveries of new nonlinear phenomena, and of stationary and nonstationary vibrations regimes with different kinetic parameters of the dynamical system (see Refs. [2] – [7]). However, even nowadays many researchers pay attention to this problem again.

Chaotic clock models, as well as original ideas on a paradigm for noise in machines were presented by F. Moon (see ref. [11]): "*All machines exhibit a greater or lesser amount of noise. The question arises as to whether a certain level of noise is natural or inevitable in a complex assembly of mechanical or electromechanical devices*? " In the cited paper, the nature of noise or chaos in a specific class of complex multi-body machines, namely the clock was examined.

For examining natural clocks of reductors (power transmission), as well as source of nonlinear vibrations and noise in its dynamics, it is necessary to investigate properties of nonlinear dynamics, and phase portraits, as well as structures of homoclinic orbits, layering and sensitivity of this layering of homoclinic orbits and bifurcation of homoclinic points.

In the paper mass moment vectors introduced by author (see Refs. [2], [5] and [10]), are used to present a vector method for the analysis of kinetic parameter dynamics of coupled rigid rotors with deviational properties of mass changeable distribution and with coupled rotations.

Following up the idea of Mossera that the distance between trajectories be measured maintaining different time scales or "clock" with which time is measured along each motion, Leela (see Ref. [12]) defines the new concepts of orbital stability in term of given topology of the function space. Leela's paper pointed out the different kind of clock. Perfect clock corresponds to stable system dynamics, entire clocks space corresponds to the chaotic topology and chaoticlike dynamics of the system.

Forced Nonlinear Dynamics Differential Equation of Heavy Coupled Rotors in the Field of Turbulent Damping

Our attention is focused on the motion of the representative point on the phase trajectory in the phase plane of forced and free nonlinear dynamics of multi-step coupled heavy rigid rotor dynamics in the field with and without turbulent damping (see Fig. 1. a*). The corresponding differential equation is in the following form (see Ref. [:

$$\ddot{\varphi}_1 + 2\delta_{1M}\dot{\varphi}_1|\dot{\varphi}_1| + \Omega_{1M}^2 \left\{ \sin\varphi_1 + \left[\sum_{i=2}^{i=M} \frac{\prod_{j=1}^{j=i} i_{j,(j+1)}}{\prod_{j=1}^{j=i}\lambda_{j,(j+1)}} \nu_{1i}^2 \sin\left(\prod_{j=1}^{j=i} i_{j,(j+1)}\varphi_1\right)\right]\right\} = h_{1M}\cos\omega t \qquad (1)$$

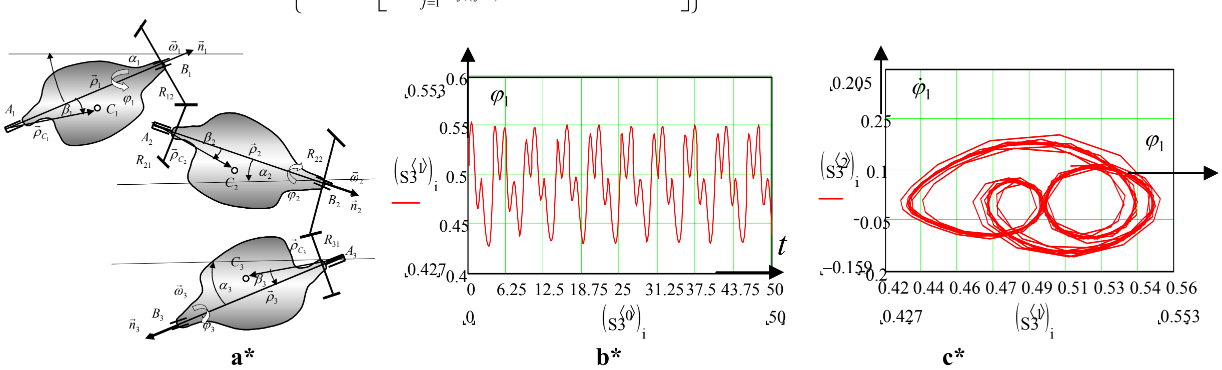
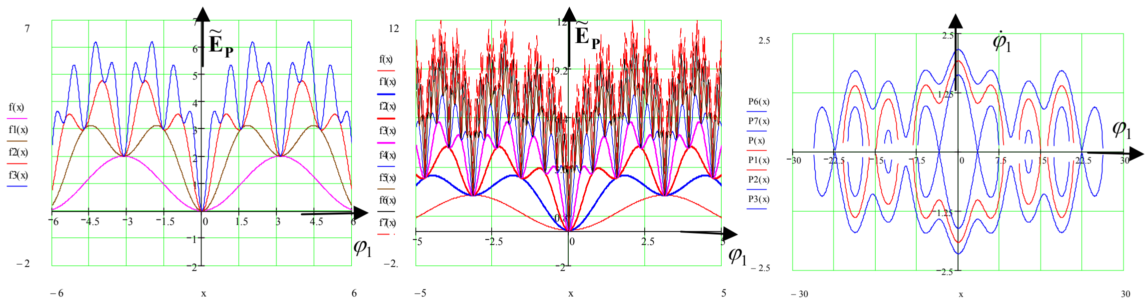


Figure 1. Three-step coupled heavy rotors (a*) forced nonlinear dynamics vizualizations.(reductor and multipliers) . b* Time-history curves; and c* Phase trajectory portraits of the forced nonlinear dynamics.

where φ_1 is generalized coordinate, Ω_{1M} , $\lambda_{j,k}$, $i_{j,k}$ kinetic parameters of the system, h_{1M} , ω and δ_{1M} parameters of external excitations, forced and damping.



a* b* c*

Figure 2. Multistep-coupled heavy rotors free nonlinear dynamics vizualizations (Multipliers).a* and b* Potential energy curves families. c* Phase homoclinic trajectories potraits (constante energy curves). Homoclinic orbits layering for conservative system.

Concluding Remarks

A numerical experiment with the use of derived analytical expressions and of MathCAD program was used to create a visualization of phase portraits of nonlinear dynamics of coupled rotors and the layering of homoclinic orbits with respect to the system parameters change.

The phenomenon of layering of homoclinic orbits with an increased number of coupled rotors is determined, so that in two coupled rotors in the earth's gravity field phase portrait a closed homoclinical orbit in the form of number eight and a trigger of coupled singularities composed of three singular points, one unstable saddle type homoclinic points and two stable centre type points appears. When the number of coupled rotors is greater then two, a greater number of homoclinic orbits layers and triggers of coupled singularities appear in series ordered a* 3, 7, 11, 15, 19....(4i-1) or b* 5, 9, 13, 17, 21,.. (4i+1); i=1, 2, 3, 4, 5,..., N. During that process in the first line (set) of coupled singularities, an odd number of unstable saddle type homoclinic points appears N(2i-1) and an odd number of stable centre type points S(2i) appears, while in the other line of coupled singularities an even number of homoclinic unstable saddle N(2i) type and an odd number of stable center type points S(2i+1) appears. With them, an appropriate number of corresponding closed homoclinic orbits that pass through homoclinic points of unstable saddle type that cut themselves, during which those with higher number of singularities envelop those with smaller number of coupled singularities.

This paper shows, on an example of coupled rotors, the correctness of the trigger of coupled singularities theorem and the existence and nonexistence of homoclinic orbits in the form of number eight theorem formulated by the author in his paper (see Ref. [10]). A theorem of layering of homoclinic orbits and of properties of those orbtis is also formulated.

The existence of the trigger of coupled singularities and closed homoclinic orbits and that their layering with respect to change of kinetic parameters of the system is the cause for the appearance of chaotic and stochasticlike system behaviour under the influence of purely periodical system excitation, as well as the appearance of sensitivity of system dynamics on small changes of starting initial conditions in the area of homoclinic points and oribits, was shown. *Than we conclude that model of reductor has set of natural clocks depending of initial conditions in free and also in forced regimes. We can point out that in this system in the forced regimes of nonlinear motion we have sensitive dependence of the initial conditions, and in this case many possible chaoticlike and stochastic lake processes appear. Than we point out direct coupling between vibrations regime, noise and clock of the system.*

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