EFFECT OF BOTTOM UNDULATIONS ON THE STABILITY OF FILM FLOW DOWN INCLINED PLANES

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Summary We study the effect of bottom undulations on the stability of a stationary film flow down inclined planes. Allowing for rather moderate bottom variations, we carry out a linear stability analysis and show how the wavy bottom affects the instability. Contrary to results for weakly undulated bottoms described in literature, where the instability is identical to that over a flat incline, we obtain an increase of the critical Reynolds number and a smaller unstable frequency spectrum with respect to the flat bottom in accordance with experimental observations.

INTRODUCTION

The instability of the steady film flow down an inclined plane has been studied in detail during the last decades [1]. Theoretical studies usually assume idealized conditions such as a perfectly flat incline and disregarding inflow and outflow disturbances. As a deviation from the perfect conditions we consider how moderately wavy bottom variations affect the linear stability of the two-dimensional film flow down inclined planes. As a starting point of our stability analysis, we take the steady flow of thin films over moderately undulated bottoms [2], [3]. By considering moderate bottom variations, our analysis differs from Tougou's who carried out a linear stability analysis for the flow over weakly wavy bottoms [4]. He found that the instability was identical to that over a flat incline without any change of the critical Reynolds number. However, recently, in experiments with rectangular corrugations a higher critical Reynolds number than that for a flat incline has been measured [5]. We show that, different from Tougou's result, the wavy bottom does affect the instability. The bottom waves stabilize the steady flow and increase the critical Reynolds number. The character of the long-wave instability remains but the bottom undulation yields a narrower unstable band than for a flat incline [6].

STABILITY ANALYSIS

Considering two-dimensional flow of a thin film down a sinusoidal bottom of moderate waviness, we carry out a linear stability analysis for the stationary flow using the local coordinate system and the scaling applied in [3]. Especially the perturbation parameter of the linear stability analysis is the same as for the analysis of the stationary flow, i.e. the product of the film thickness and the wave number of the bottom contour. Besides the perturbation parameter, the flow is described by the Reynolds number, the cotangent of the inclination angle, an inverse Bond number, and the waviness of the bottom contour. The Reynolds number is constructed with the film thickness and the maximum velocity of the stationary film flow over a flat incline. The inverse Bond number takes into account the effect of surface tension over the scale of the bottom wavelength: $1/Bo = 1/(Bo^{*}\sin\alpha) = (2\pi l_{Ca}/\lambda)^2/\sin\alpha$, where α, λ, l_{Ca} are the inclination angle, wavelength of the bottom, and capillary length of the fluid, respectively. The waviness is the product of amplitude and wave number of the bottom wave. In the analysis, all these fixed parameters are assumed to be of order one. Thus, the waviness is small enough to avoid the creation of vortices in the valley of the wavy bottom [7].

The analysis is restricted to monotonously falling bottom contours, having, on the other hand, the advantage that the stationary film can be described at leading order essentially as one flowing over a flat plane at the respective local inclination angle. This has important implications. For instance, the film thickness averaged over one bottom wave is larger than for the flat bottom inclined at the mean angle.

The results of the linear stability analysis can be summarized as follows: The instability remains to be one of long-wave type. However, the unstable band is narrower. The instability can be expressed in local terms: The critical Reynolds number is then given in terms of the mean of the local hydrostatic pressure

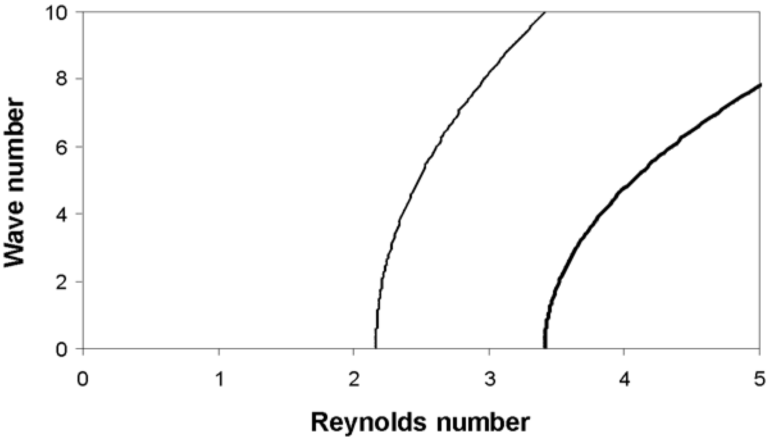

Figure 1: Neutral stability curves. The thin and thick lines represent the marginal curves for the flat plane and a wavy bottom, respectively. Waviness: 2π·0.05, inclination angle: 30°, inverse Bond number: 0.01.

contribution and the local capillary pressure contribution. The hydrostatic pressure contribution, expressed by the mean of the cotangent of the local inclination angle, is always larger than that of the flat bottom. The same holds for the capillary pressure contribution that yields an additional term, also for the long-wavelength limit, due to the bending of the free surface by the bottom contour. An example of the neutral curve comparing the undulated bottom to the flat one is reproduced in Figure 1.

Figure 2 shows that the increase of the critical Reynolds number can be quite important. It not only augments with the waviness of the bottom contour but also with decreasing inclination angle. The latter is due to both, hydrostatic and capillary pressure contributions. Furthermore, the figure indicates why considering weakly wavy bottom contours did not yield any deviations from the results obtained for a flat plane.

CONCLUSIONS

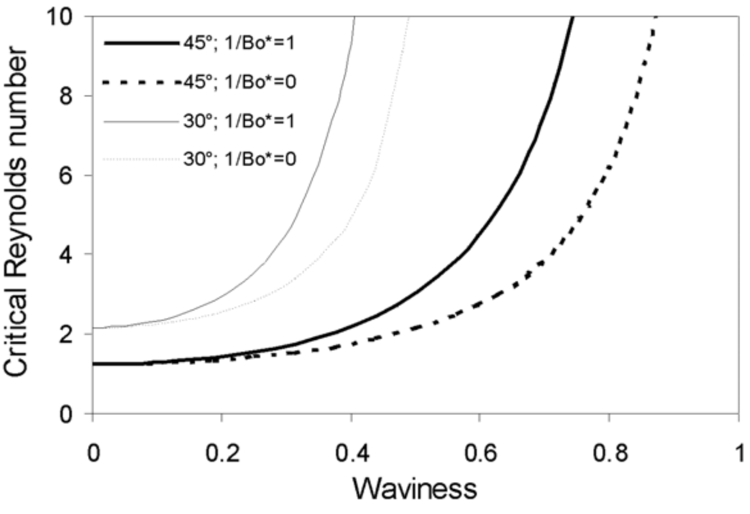

Figure 2: Influence of the waviness and the Bond number on the critical Reynolds number. Inclination angle: 45°.

We carried out a linear stability analysis for the film flow over wavy bottoms. The analysis is restricted to thin films over monotonously falling bottoms, and the Reynolds number and the inverse Bond number are to be of order one or smaller. Within these restrictions, the critical Reynolds number for long surface waves is higher than that for a flat plane. Surface tension alters the minimum critical Reynolds number. It deviates from the flat plane solution with increasing waviness and with decreasing inclination angle. The instability occurs at long wavelength as in the case of a flat bottom. Furthermore, the bottom undulation yields a narrower frequency spectrum than for a flat incline.

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