DYNAMIC CRACK ANALYSIS UNDER THERMAL SHOCK

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Summary

A boundary element method using Laplace transform in time domain is developed for the analysis of fracture mechanics considering transient coupled thermoelasticity problems with relaxation times in two dimensional finite domain. The dynamic thermoelastic model of Lord and Shulman are selected for showing finite thermal propagation speed. Thermal dynamic stress intensity factor for mode I is evaluated using different relaxation times.

Introduction

The classical theory of thermoelasticity assumes that the thermal disturbances propagate at infinite speeds. This prediction is a physically unacceptable situation. There is a class of problems dealing with impingement of a high intensity energy source on the surface of a structure that needs special consideration of the thermal wave model. The wave model in the heat transport process becomes even more important if some irreversible physical processes, such as crack or void initiation and propagation in the solid, occur in the radiant duration. In these cases the orientation of crack initiation or crack propagation, for example, must be predicted according to the thermal wave model. In the classical study of thermoelastic crack problems, the theoretical solutions are available only for very few problems in which cracks are contained in infinite media under special thermal loading conditions. For example Lee and Sim [1] solved the problem of a surface cracked infinite strip under sudden conductive cooling and evaluated the mode-I thermal shock stress intensity factor (TSSIF) using Bueckner's weight function method. Katsareas et al. [2] used a boundary-only element method to compute shock stress intensity factors for a surface cracked infinite strip and a finite edge cracked plate. They considered uncoupled quasi-static thermoelasticity. Recently Hosseini-Tehrani et al. [3] present a boundary element formulation for the dynamic crack analysis Considering coupled theory of thermoelasticity. This paper presents a boundary element formulation for the crack analysis Considering the LS theory of thermoelasticity. In this work the body is exposed to a thermal shock on its boundary and the resulting thermal stress waves are investigated through the coupled thermoelastic equations. The discretized forms of the equations are obtained by the approximation of boundary variations by quadratic elements.

GOVERNING EQUATIONS

A homogeneous isotropic thermoelastic solid is considered. In the absence of body forces and heat generation, the governing equations for the dynamic coupled generalized thermoelasticity in time domain based on the LS theory are written as

\[ (\lambda + \mu) u_{ij,yy} + \mu u_{ij,yy} - \gamma T_{ij} - \rho \ddot{u}_i = 0, \quad k T_{ij} - \rho c_v \ddot{T} - \rho c_p \ddot{T}_c - \gamma T_c (t_0 u_{ij,yy} + u_{ij}) = 0 \]  

where a dot indicates time differentiation and the subscript \( i \) after a comma refers to partial differentiation with respect to \( x_i \) (\( i=1,2 \)), and \( \lambda, \mu, u_i, T, T_0, k, c_v, c_p, \) and \( t_0 \) are Lame's constant, the components of displacement vector, mass density, absolute temperature, reference temperature, thermal conductivity, stress-temperature modulus, specific heat at constant strain and, relaxation time proposed by LS theory respectively. Equations (1) can be rewritten in matrix from as

\[ L_q U_j = 0 \]  

In order to drive the boundary integral problem, weak formulation of the differential equation set (2) for the fundamental solution tensor \( V_{ik}^* \) may be used

\[ \int_\Omega (L_q U_j) V_{ik}^* d\Omega = 0 \]  

The detail mathematical formulations may be found in Hosseini-Tehrani et al. [3]. The stress intensity factor may be determined either from nodal traction or from the crack opening displacement (COD). To evaluate \( K_i \), the quarter point singular element at the crack tip is used. If \( v_B \) at \( r=l/4 \) is considered ( \( l \) is the length of the singular element at the crack tip) \( K_i \) may computed as

\[ K_i = \frac{\mu}{4 - 4\nu} \frac{2\pi}{l} \sqrt{\frac{v_B}{l}} \]  

RESULTS AND DISCUSSION

Structures covered with coatings and lining may be subjected to sever loading conditions. The load might be applied in the form of mechanical and/or thermal shocks. If the period of shock duration is small enough compared to the first natural frequency, then the dynamic thermoelasticity may be important. Considering an infinite strip shown in Fig. (1),
initially subjected to a uniform temperature $T_0$ with an edge crack perpendicular to its top surface. The strip is rapidly cooled by conduction at its upper surface $x_2=0$, whereas the bottom surface $x_2=W$ is insulated. This is a mode I crack opening problem. The crack edges assumed to be thermally insulated. Due to the symmetry about the $x_2$ axis, only half of the strip is discretized. The boundary element model is presented in Figure 1(b), for a crack depth of $a^*=0.05$ and $L/W=1$, where $a^*=a/W$. $K^*_I$ is defined as the dimensionless TDSIF, which for plain strain condition is

$$K^*_I = K_I (1 - v) \sqrt{\frac{E}{\rho c W^2}} \left[ T - T_0 \right]$$

where $t^*=Kt/(\rho c W^2)$ is the dimensionless time known as Fourier number. Validation of this method is shown in [3] for classical theory.

Figure (2) shows crack intensity factor variation versus dimensionless time $t^*$, while $t^*=1$ means $9.3E-12$ sec. In Fig. 2 comparison between conventional uncoupled and LS theory is done. In this figure curves are plotted for three different conditions. $C=0$ represent conventional uncoupled theory of thermoelasticity, in this case $K^*_I$ is increased instantaneously after the application of the sudden cooling. At time $t^*=0.002$, the thermoelastic wave stress reaches the tip of the crack. This cold shock produces tensile stress in the $x_2$-direction and due to the effect of the Poisson's ratio, compressive stress is produced in the $x_1$-direction. This phenomena results into crack opening and thus $K^*_I$ increases by time, as shown in Fig. (2). When $t_0=0.64$, the speed of propagation of the thermal wave is $C_1=1.25$ and the speed of the propagation of the stress wave is $C_2=1$ (dimensionless). In this condition at $t^*=0.00175$ when cold wave reaches to the crack tip, crack opening begins. At $t^*=0.002$ when stress wave reaches to crack tip $K^*_I$ variation takes place. For $t_0=1.5625$ the speed of propagation of thermal wave is $C_1=0.8$ and the speed of propagation of the stress wave is $C_2=1$ (dimensionless). In this condition at $t^*=0.0025$ when cold wave reaches to the crack tip, crack opening begin, but stress wave that was already reached to crack tip has a decreasing effect on $K^*_I$.

**CONCLUSIONS**

A boundary element method and Laplace transform in time domain are developed for the analysis of fractured planar bodies subjected to thermal shock type loads. The important results of this study are: The appropriate time scale in which the effect of inertia term is observed is considered and the importance of inertia term is shown. In conventional theory of thermoelasticity, $K^*_I$ has risen instantaneously after applying cold shock but in LS theory $K^*_I$ rises when temperature or stress wave reaches to crack tip. The fracture analysis due to thermal shock using LS theory of thermoelasticity shows more fluctuation on $K^*_I$’s curve versus time.

**References**
