# PLASTIC PROPERTIES IDENTIFICATION WITH PLURAL SHARP INDENTERS

Norimasa Chiba, Nagahisa Ogasawara

National Defense Academy, Yokosuka, Kanagawa 239-8686, Japan

<u>Summary</u> An identification method for elastic-plastic material constants that obey the power-law hardening rule, from a couple of instrumented sharp micro-indentation tests, is proposed. This method utilizes plural triangular pyramid indenters with different apex angles, and is based on a similarity function that is determined from the 3D-FE calculation. A physical explanation of the similarity function expression is given. An experimental validation for this method with actual metal is also shown.

## SIMILARITY RELATION FOR POWER-LAW HARDENING MATERIAL

Indentation load, *P*, when indented into a half infinite solid, can be expressed as  $P=Ch^2$ , known as Kick's law, where *h* and *C* are the indented depth and the loading curvature, respectively. We assume the indented material is elastic powerlaw hardening plastic (Fig. 1), with Young's modulus *E*, yield strength  $\sigma_Y$ , and hardening exponent *n*. Then, the loading curvature *C* can be expressed in Eq. (1), using a dimensionless function  $\Pi$  and an arbitrary stress measure  $\sigma_r[1,2]$ . In Eq. 1, *E*\* means reduced Young's modulus, which is the only elastic property determined from indentation, and  $\theta$  half angle of the apex of the conical indenter. The reduced Young's modulus *E*\* is expressed by  $1/E^*=(1-v^2)/E$ , if we assume the indenter to be rigid. In the actual micro-indentation test, a triangular pyramid indenter is used. However, analytical results used in this paper have been obtained on the basis of 2D axisymmetricity. Therefore, we use an equivalent cone that has the same *h*-*A* relation with the triangular pyramid (Fig. 2). Dao, et al. showed from their extensive FE calculation that if we choose an appropriate value of  $\sigma_r$ , then the relation, shown in Eq. 2, holds for a fixed cone angle  $\theta$ of the indenter [2]. This means that the value of  $\Pi$  for the appropriate  $\sigma_r$  is independent of the hardening exponent *n*. We also verified that Eq. (2) holds for 3 angles of cones from 3D FE calculation. These results are described later in this paper.



Fig. 1 Power-law hardening stress-strain relation

Fig. 2 Triangular pyramid and equivalent cone

#### EXPRESSION OF $\Pi$ AS AN INTERPOLATION BETWEEN TWO EXTREMES

If we plot the relation of Eq. (2) in the form of  $C/\sigma_r$  versus  $E^*/\sigma_r$ , then we may consider that the relation has two extremes: one is the rigid/perfectly plastic solution, where  $E^*$  tends to be infinite, and  $\sigma_r$  represents a finite value of yield strength  $\sigma_Y$ . The other extreme is the elastic solution, where  $E^*$  has a finite value, and  $\sigma_r = \sigma_Y$  tends to be infinite. Lockett obtained an axisymmetric slip-line field solution indented by a rigid cone [3]. The solution assumed smooth contact between the indenter and the solid, and the solution method has been extended to take account of the friction effect [4]. Using Lockett's solution, the dimensionless function  $\Pi$  for the rigid/perfectly plastic extreme is expressed in Eq. (3), where *k* and  $f(r_c)$  stand for the yield strength in shear mode, and the piling-up height, respectively. Using elastic solution [5], the function  $\Pi$  for the elastic extreme is expressed in Eq. (4). Then, we may write the function  $\Pi$  for elastic plastic region as Eq. (5), which is a simple interpolation between the two extremes.

$$\Pi = \frac{P}{\sigma_{\rm Y} h^2} = \left(\frac{P}{k\pi}\right) \frac{\pi}{2(\cot\theta - f(r_C))^2} \equiv a_p \tag{3}$$
$$\frac{1}{\Pi\left(\frac{E^*}{\sigma_r}, \theta\right)} = \frac{1}{a_k^{(\theta)} \times \left(\frac{E^*}{\sigma_r}\right)} + \frac{1}{a_p(\theta)} \tag{5}$$

$$\Pi = \frac{P}{\sigma_r h^2} = \left(\frac{2}{\pi} \tan\theta\right) \frac{E^*}{\sigma_r} \equiv a_e \frac{E^*}{\sigma_r}$$
(4)

# DIMENSIONLESS FUNCTION $\Pi$ determined from finite element calculation

We carried out 3D FE calculation for three angles  $\phi$  of pyramid indenters, shown in Fig. 2(c). The material parameters chosen for each apex angle  $\phi$  are: *E* ranging from 10 to 210 GPa,  $\sigma_{\rm Y}$  from 30 to 3000 MPa, and *n* from 0.0 to 0.6. We confirmed that Eq. (2) holds if the appropriate value of representative strain  $\varepsilon_r$  is chosen for each indenter angle  $\phi$ . This is shown in Fig. 3. The representative stress  $\sigma_r$  was determined from representative strain  $\varepsilon_r$  on the corresponding  $\sigma$ - $\varepsilon$  curve. The three values of representative strain determined from the calculation for each indenter angle are shown in Fig. 4.



Fig. 3 Dimensionless function  $\Pi$  constructed from FE calculation for three indenter angles

## **EXPERIMENTAL VALIDATION**

We carried out indentation tests with the three different indenters on two types of copper polycrystal specimens, work hardened and annealed. Fig. 5 shows the result. The symbols on the  $\sigma$ - $\varepsilon$  curve represent the results determined from our plural indenter method. The lines show the tensile test results obtained from bulk material. It can be concluded that our method provides a satisfactory result.



Fig. 4 Representative strains for different indenter angles



Fig. 5 Points determined from indentation

### References

- [1] Cheng, Y-T., et al., J. Appl. Phys. 84: 1284-1291, 1998.
- [2] Dao, M., et al., Acta Mater. 49: 3899-3918, 2001.
- [3] Lockett, F. J., J. Mech. Phys. Solids 11: 345-355, 1963.
- [4] Chitkara, N. R., et al., Int. J. Mech. Sci. 34: 849-862, 1992.
- [5] Sneddon, I. N., Proc. Cambridge Phil. Soc. 44: 492-507, 1948.
- [6] Bucaille, J. L., et al., Acta Mater. 51: 1663-1678, 2003.