NON-LINEAR STOCHASTIC VIBRATION PROBLEMS FOR THE PLATES WITH TIME-DEPENDENT PARAMETERS

Victor Z. Gristchak*, Valentin V. Lysenko*
*ZSU, Department of Applied Mathematics, Zhukovskogo 66, UA 69-600, Zaporizhzhja, Ukraine,
ID 10656, grk@zsu.zp.ua

Extended Summary

The correlation analysis of behaviour of non-linear systems with time-depended parameters at random loading deals with the significant difficulties in comparison with an analysis of linear systems with constant parameters.

In this paper the new hybrid asymptotic technique that combines the perturbation, WKBJ methods and Hamilton variational principle for the solution of non-linear stochastic problem for the plates with time-dependent parameters is presented. In order to describe the hybrid technique in detail, let us consider non-linear differential equation of second order with time-dependent coefficients, which contains perturbation parameters (usually large or small) and which is used while solving a number of applied problems:

$$f''(t) + \omega_0^2 \cdot \varphi(t) f(t) + \alpha \cdot \left(P(t) f^2(t) + Q(t) f^3(t)\right) = \gamma(t), \qquad (1)$$

where ω_0, α – are parameters of frequency of linear system and degree of non-linearity. If the plate is compressed along one direction by static loads and as external force a time-dependent distributed cross loading is enclosed to a plate, then in the equation (1)

$$\omega_0^2 = \frac{\pi^4 m^4 \left[1 + \left(\frac{n\lambda}{m}\right)^2\right]^2}{12\lambda^2(1-\mu^2)} \left(\frac{v h_0}{ab}\right)^2, \quad \alpha = \frac{h_0}{a},$$

where: $v = \sqrt{E_0/\rho}$ – is a speed of longitudinal wave propagation in a material of a plate; *f* – is a function of a displacement; $\lambda = \frac{a}{b}$. Let's search for the analytical solution of the initial equation (1) on finite interval of time $[t_0, T]$. The function *f* is subjected to some initial conditions.

At step one (external asymptotic), we use the perturbation method to find an approximate solution for *f* in the form

$$f(t) = \sum_{j=0}^{\infty} \alpha^j f_j(t), \qquad (2)$$

where $f_j(t)$ (j = 0, 1, ...) – unknown functions of time. We have to substitute the expansion (2) into equation (1) and equate the coefficients of the same powers of α in the resulting equations:

$$\begin{aligned} &L_0 f_0(t) = \gamma(t), \\ &L_0 f_1(t) = -\left(P(t) f_0^2(t) + Q(t) f_0^3(t)\right), \\ &\ldots, \end{aligned} \qquad (3)$$

where $L_0 = \frac{\partial^2}{\partial t^2} + \omega_0^2 \varphi(t)$ – is a linear differential operator.

At step two (internal asymptotic), we have to find the approached solution of the linear homogeneous differential equation like

$$L_0 f_{0_{Hom}}(t) = 0 \qquad (4)$$

with the help of WKBJ method. By replacement $f_{0_{Hom}}(t) = \exp\left(\int_{t_0}^{t} \psi(\theta) d\theta\right)$, we shall receive the differential equation concerning unknown function of time $\psi(t)$:

$$\varepsilon^2 (\psi'(t) + \psi^2(t)) = -\varphi(t), \qquad (5)$$

where $\varepsilon = 1/\omega_0$. According to a WKBJ method the approached solution of the equation (5) we shall present as

$$\psi(t) = \sum_{k=0}^{\infty} \varepsilon^{k-1} \psi_k(t), \tag{6}$$

where $\psi_k(t)$ – is an unknown function of time (k = 0, 1, ...). We have to substitute the expansion (6) into equation (5), equate the coefficients with the same order of ε in the resulting equation and solve a sequence of equations for each of the unknown $\psi_k(t)$ (k = 0, 1, ...).

At step three (internal hybrid solution) of the hybrid method, we use a subset of functions $\psi_k(t)$ (k = 0, 1, ...) of WKBJ-expansion (6), determined at step two, in a WKBJ type expansion, named as hybrid:

$$\psi_H(t) = \sum_{k=0}^{M-1} \lambda_k \psi_k(t). \tag{7}$$

Here the M – is the order of internal approach; λ_k (k = 0, ... , M – 1) represents new unknown "amplitudes" of known functions $\psi_k(t)$ (k = 0, ..., M – 1). Then, it is possible to obtain the hybrid solution of the linear homogeneous differential equation (4) and hybrid solutions of each equation of system (3).

At step four (external hybrid solution) we use a subset of hybrid functions $f_{j_H}(t)$ (j = 0, 1, ...) of perturbation-expansion (2), in a perturbation type expansion, named as hybrid too:

$$f_H(t) = f_{0_H}(t) + \sum_{j=1}^{N-1} \delta_j f_{j_H}(t). \tag{8}$$

Here the N – is the order of external approach; δ_j (j = 1, ... , N – 1) represent unknown parameters. The principle of "optimal" determining of the λ_k (k = 0, ..., M – 1) and δ_j (j = 0, ..., N – 1) is based on the Hamilton variational principle

$$\int_{t_0}^{T} (P + K - A)dt \to 0, \tag{9}$$

where P – is a potential energy, K – is a kinematical energy of the plate and A – is a work of internal and external forces.

For the solution of non-linear stochastic vibration of plates the moment functions of output process are determined by averaging a perturbation-expansion (8). So, for second-order moment function we obtain:

$$\langle f_H(t_1) f_H(t_2) \rangle = \langle f_{0_H}(t_1) f_{0_H}(t_2) \rangle + \delta_1 \langle f_{0_H}(t_1) f_{1_H}(t_2) + f_{0_H}(t_2) f_{1_H}(t_1) \rangle + \ldots \tag{10}$$

In order to reduce higher order moment function of the zero-order approximation $f_{0_H}(t_1)$ we shall use expressions binding higher order moment functions with lower ones, which are correct for normal processes (input and output processes are assumed to be centralized). And then, it is possible to obtain a hybrid correlation function of a displacement of the plate.

It is shown, that hybrid results are more accurate than the classical asymptotic perturbation-WKBJ-solution.

References

[1] Noor A.K., Peters J.M.: Reduced basis technique for nonlinear analysis of structures. *Proc. of the AIAA-ASME-ASCE-AHS 20th Structures*, Structural Dynamics and Materials conference, St. Louis, USA, 1979. – P. 116 – 126.

[2] Geer J.F., Andersen C.M.: A hybrid peturbation-Galerkin technique for differential equations containing a parameter. *Applied Mechanics Reviews*. – 1989. – ¹42(11). – P. S69-S77.

[3] Gristchak, V.Z., Dmitrijeva, Ye.M.: A hybrid WKB-Galerkin method and its application. *Technishe Mechanik*, 1995. – ¹3. – Đ. 281-294.

[4] Gerasimov T.S., Lysenko V.V.: A hybrid approximate solution technique and modal expansion approach with application to forced vibrations of non-homogeneous shells and plates. *Proc of the 7th Conference "Shells structures, theory and applications"*, Gdansk-Jurata, Poland, 2002. – P. 85-86.