REISSNER-MINDLIN-VON KÁRMÁN TYPE PLATE MODEL FOR POSTBUCKLING ANALYSIS OF LAMINATED COMPOSITE STRUCTURES

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Summary Postbuckling behaviour of laminated composite structures using Reissner-Mindlin-Von Kármán type plate model is considered. The potential energy functional where the linearized strain tensor has been replaced by nonlinear Von Kármán model for large deformation is formulated. The stabilized MITC-technique is used to discretize the problem by finite elements. Load-displacement behaviour of the structure as well as the failure prediction and the identification of the critical areas of the model are illustrated with numerical examples.

REISSNER-MINDLIN-VON KÁRMÁN MODEL FOR PLATE BENDING

Due to their unique strength and stiffness properties, and their dimensional stability under hygrothermal loads, fiber-reinforced composites are widely used in many weight-critical industries like aerospace, automotive, and marine industry. In practical engineering problems, the buckling load often becomes a critical design criterion because of the thin-walled nature of the strength-optimized structure. Composite materials usually do not exhibit large yielding deformations before failure and therefore do not allow stresses to be redistributed in the postbuckling region [5]. Hence, a more detailed analysis of the postbuckling region is needed to fully predict the behaviour of composite structures.

In this work, we consider nonlinear analysis of laminated composite structures subjected to large deflections. The model is based on a facet approximation of the midsurface and a Reissner-Mindlin-Von Kármán type plate equations. We formulate the potential energy functional where the linearized strain tensor has been replaced by nonlinear functions, i.e., the Von Kármán model for large deformation. The fourth-order tensors connecting membrane stress resultants and bending moment resultants to membrane strains and curvatures, respectively, and the coupling one for in-plane and flexural behaviour as well as the second-order tensor connecting transverse shear forces to transverse shear strains are defined according to the Classical Lamination Theory.

In the FE-implementation we use linear triangular elements [3]. The transverse shear strains are discretized by the stabilized MITC-technique of Brezzi, Fortin, and Stenberg [2]. The normal rotation ("drilling rotation") is introduced as an auxiliary function and discretized by the stabilized method of Hughes and Brezzi [4]. The material principal orientation is defined by three rotations for each element.

Table 1 AS4/3501-6 carbon-epoxy ply mechanical properties.

E_1 [GPa]	E_2 [GPa]	G_{12} [GPa]	G_{31} [GPa]	ν_{12}	ν_{23}
139.3	11.1	6.0	6.0	0.3	0.4

X_t [MPa]	X_c [MPa]	Y_t [MPa]	Y_c [MPa]	S [MPa]
1950.0	1480.0	48.0	200.0	79.0

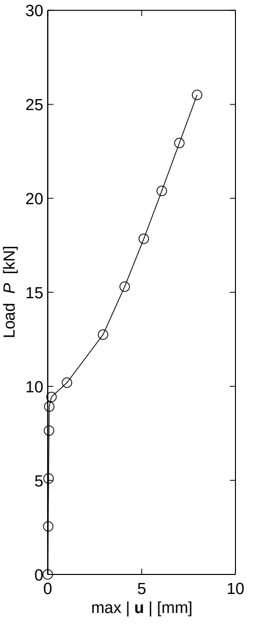

Figure 1 Compressive load-displacement behavior of the [2(0)/2(+45)/0]SE plate.

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In the laminate First Ply Failure (FPF) analysis, we employ a generalized method for solving the laminate failure margin with an iterative solver. Failure criteria are handled as black boxes whose internal formulation has no effect on the solution procedure.

NUMERICAL EXAMPLE

The shell model has been recently implemented in the ELMER software package [1] developed at CSC for solving PDEs by FEM. In the numerical example shown here we consider a rectangular plate of 255×255 mm composed of layers having the nominal ply properties of Hercules AS4/3501-6 carbon-epoxy [5] indicated in Table 1. The laminate lay-up is [2(0)/2(+45)/0]SE with $N = 10$ layers and the plate is subjected to compressive load at one end of the plate. The boundary conditions are considered to be clamped on all four sides of the plate. The element mesh contains 3200 linear triangular elements. In computation we have used the stabilation factor $\alpha = 0.2$.

The results show the minimum energy solution corresponding to the first buckling mode of plate. The critical linear buckling load is $P = 9425$ N. In Figure 1 the compressive load-displacement behavior of the plate in terms of max $|\mathbf{u}|$ is represented. Figure 2 illustrates the reserve factor value as a function of compressive load for maximum strain and Tsai-Wu failure criteria with logarithmic scales. The failure analysis results can also be visualized by using the inverse reserve factor to identify the critical areas of the FE model.

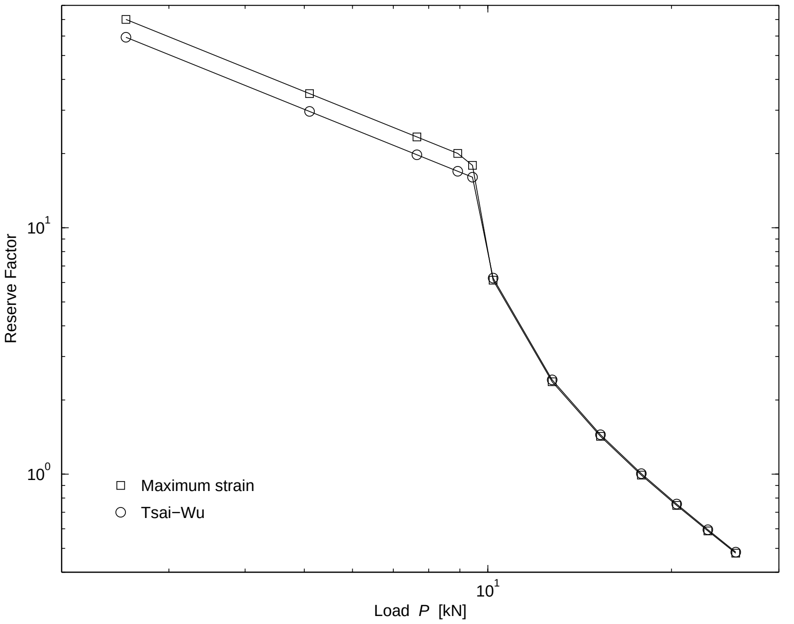

Figure 2 Reserve factor value as a function of compressive load with logarithmic scales.

Acknowledgement

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