ELASTIC WAVE PROPAGATION DEVELOPMENT FOR STRUCTURAL HEALTH MONITORING

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Abstract This paper is a personal perspective of structural health monitoring technology and its applications as see from a current literature and projects. The proposed approach deals with the spectral finite element analysis method as a means of solving the wave propagation problems in structures. The change of the wave propagation process due to a damage appearance is examined by comparing the differences between the responses from damaged and undamaged structures. The influence of the damage growth for the wave propagation is also analysed. The differences in the propagating waves allow indicating the damage location and size in a very precise way.

Keywords: Damage detection, structural health monitoring

1. Introduction

Undetected damage in buildings, bridges, aerospace structures, containers and other large structures will develop if undetected and in some cases result in the collapse of the whole structure. This may have costly disastrous consequences for people and the environment including deaths, injuries, fires, contamination, economic losses, legal costs, etc. It has been found that in a lot of cases a small–undetected damage (crack, plastification, delamination) has been the cause for the collapse.

Damage in structures should be detected at an early stage, before it can cause any problems and so that proper decisions for repair and/or part replacement can be taken.

The most popular methods used for non–destructive evaluation are: visual inspection, eddy current, X–ray, ultrasound, strain gauge, and thermal contours. Unfortunately, most of those methods have significant disadvantages. Usually, they are quite impractical for monitoring large, complex structures.

Several of NDT methods being currently developed. The most promising approaches are based on detecting: acoustic emissions (AE), strain variations in optical fiber sensors (OFS), vibration–based methods (VBM), and structural health monitoring (SHM).

This paper is focused on structural health monitoring technology and its applications in aerospace and civil engineering. The investigated damage detection system is based on the known fact that material discontinuities affect the propagation of elastic waves in solids. The change in material characteristics, such as a local change in stiffness or inertia caused by a crack or material damage, will affect the propagation of elastic waves and will modify the received signals.

Better understanding of phenomena associated with propagation of elastic waves in structural elements is very important from both the theoretical and the practical point of view.

In recent years significant progress has been evident in this particular field of applications.

2. Structural Health Monitoring Technology

SHM (Structural Health Monitoring) technology is the technological platform for a new maintenance philosophy. SHM technology works with a built–in sensor network on a structure. These sensors provide information regarding the condition and damage state of the structures as they age.

A new philosophy based on SHM technology creates feedback loops within the design, manufacturing, and maintenance procedures by providing additional knowledge about a specific design performance, material quality, and structure condition respectively.

Changes in propagation of elastic waves when observed in structural elements can be used for the purpose of damage detection or identification of material parameters within those elements.

Elastic waves are generated and sensed by an array of transducers either embedded in, or bonded to, the surface of the structure. Wave frequencies associated with the highest detection sensitivity depend, among others, on the type of the structure, the type of material, and the type of the damage. Figure 1 presents a general concept of this technology.

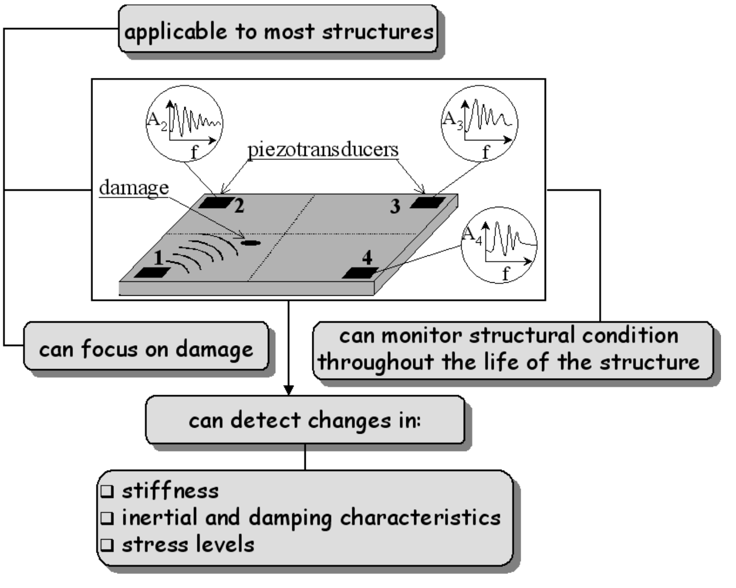

Figure 1. Build in sensor network on a plate.

3. Spectral Finite Element Technique

Among many frequency domain methods, the spectral method is found to be suitable due to its adaptation of displacement based FE technique and therefore enabling one to handle arbitrary skeletal structures.

The SFEM approach is based on exact solution to governing Partial Differential Equations (PDE) in the frequency domain. This exact solution is used as interpolating function for spectral element formulation. As a consequence, relatively small number of elements can be used for modelling without loosing the accuracy. The solution is obtained in terms of generalised displacements, subsequent calculations for velocity, acceleration, strain and stress for any applied load can then be found with relatively inexpensive postprocessing calculations. The spectral element method directly computes a structure's frequency response function and in this manner gives additional information that bridges the gap between modal methods based on free vibrations and time reconstruction based on direct integration. This is particularly useful for wave propagation modelling.

Figure 2 shows a flow diagram for the basic algorithm to propagate a wave. The governing wave equations are first transformed from the time domain to the frequency domain using a discrete Fourier transform

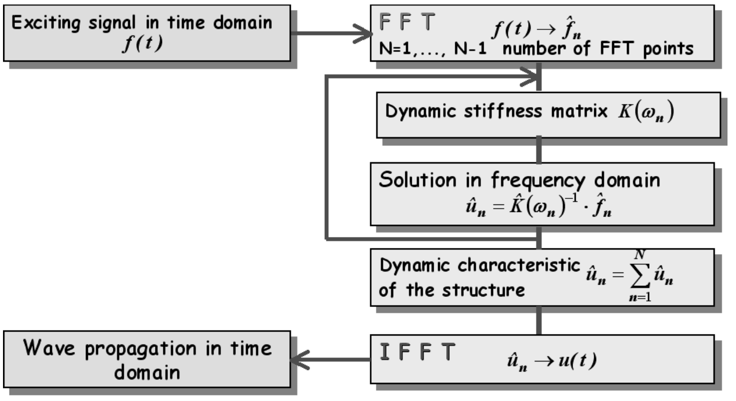

Figure 2. Flow diagram for the wave reconstruction program.

(DFT). For computational implementation we use the FFT algorithm built into the FE code.

In a simple form, the solution to a wave propagation problem can be represented as by Doyle [1]:

$$\hat{u} = \sum_n \hat{P}_n \left\{ \hat{G}_1 \left(k_{1n} x \right) + \hat{G}_2 \left(k_{2n} x \right) + \ldots \right\} \exp\left(i \omega_n t \right)$$
$$= \sum_n \hat{P}_n \hat{G}_1 \left(k_{mn} x \right) \exp\left(i \omega_n t \right) \quad (1)$$

where: $\hat{G}$ is the analytically derived transfer function of position x, $\hat{G}$ has different numerical values at each frequency. $\hat{P}_n$ is the amplitude spectrum and is known from input conditions or from measurements. $P_n G$ is recognised as the Fourier transform of the solution.

The spatial variation is semi-explicitly obtained by solving the characteristic equation in k-space. This results in a complex shape function matrix representing the linear superposition of all the wave modes. Also, the complex dynamic strain-displacement matrix and the dynamically consistent force vector can both be derived. Following the conventional FE method, the complex dynamic stiffness matrix is then formed, and this is exact. This makes the proposed SEM an efficient model suitable for use within the framework of an automated FE method rather than a trade-off for broadband wave propagation analysis. But the fundamental difference from the conventional FE method is that all the spectral amplitudes that correspond to elemental nodal variables are evaluated at each frequency step (FFT sampling points) instead of pseudo-static variables evaluated at each time step or at each eigenfrequency. The global system is solved for unit spectral amplitude of the applied load history at each FFT sampling frequency. Therefore, computation in this way of the frequency response function (FRF) of the global system is straightforward. The time domain response is obtained using the inverse FFT.

This paper presents a method of wave propagation, which can be used to detect small failures in rod, beam-like and plate structures.

Cracked Rod Spectral Element

A spectral rod finite element with a transverse open and non-propagating crack is presented in Fig. 3. The crack is substituted by a dimensionless spring, which flexibility θ is calculated by using Castigliano's theorem and laws of the fracture mechanics [2, 3].

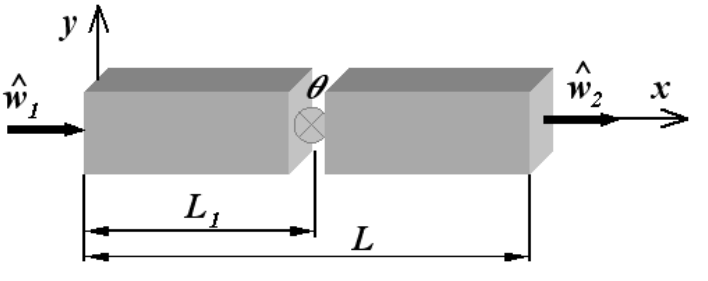

Figure 3. The model of the rod with a transverse open and not propagating crack simulated by elastic hinge.

Nodal spectral displacements are assumed in the following form for the left (Eq. 2) and right (Eq. 3) part of the rod:

$$\hat{w}_1(x) = A_1 \mathrm{e}^{-ik_n x} + B_1 \mathrm{e}^{-ik_n(L_1 - x)} \qquad \text{for } x \in (0, L_1)\,, \tag{2}$$

$$\hat{w}_2(x) = A_2 \mathrm{e}^{-ik_n(x+L_1)} + B_2 \mathrm{e}^{-ik_n[L-(L_1+x)]} \qquad \text{for } x \in (0, L - L_1) \tag{3}$$

where: L_1 denotes the location of the crack, L is the total length of the rod and k_n is the wave number calculated as follows:

$$k_n = \omega_n \sqrt{\frac{\rho}{E}} \tag{4}$$

where: ρ is the density of the rod material, E denotes Young's modulus and ω_n is a natural frequency.

The coefficients A_1, A_2, B_1 and B_2 can be calculated as a function of the nodal spectral displacements using the element boundary conditions [3]. Using boundary conditions can relate the coefficients A_1, A_2, B_1 and B_2 to the nodal spectral displacements. The nodal spectral forces can be determined by differentiating the spectral displacements with respect to x. Finally, using the relation between nodal forces and nodal displacements the dynamic stiffness matrix of a cracked rod spectral finite element can be evaluated. Propagation of elastic waves in rods is presented in [3].

Cracked Timoshenko Beam Spectral Element

A spectral Timoshenko beam finite element with a transverse open and non–propagating crack is presented in Fig. 4. The length of the element is L, and its area of cross–section is A. The crack is substituted by a dimensionless spring, which flexibility θ is calculated by using Castigliano's theorem and laws of the fracture mechanics [2, 4].

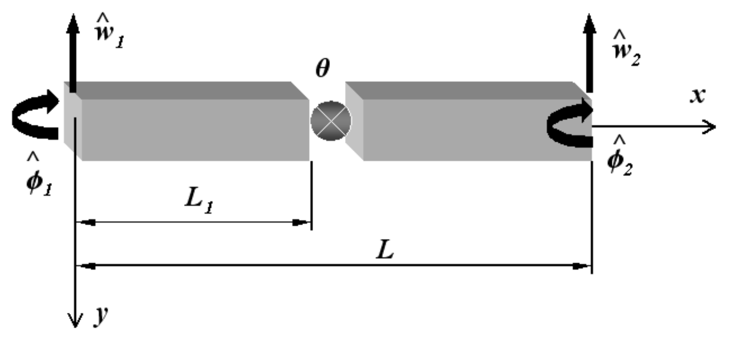

Figure 4. The model of the Timoshenko beam with a transverse open and not propagating crack simulated by elastic hinge.

Nodal spectral displacements $\hat{w}$ and rotations $\hat{\phi}$ are assumed in the following forms, for the left (Eq. 5) and right (Eq. 6) part of the beam:

$$\hat{w}_1(x) = [\Phi_1]\cdot[\Psi]\cdot\{\alpha_1\} \text{ and } \hat{\phi}_1(x) = [\Phi_1]\cdot\{\alpha_1\} \text{ for } x \in (0, L_1)\,, \tag{5}$$

$$\hat{w}_2(x) = [\Phi_2]\cdot[\Psi]\cdot\{\alpha_2\} \text{ and } \hat{\phi}_2(x) = [\Phi_2]\cdot\{\alpha_2\} \text{ for } x \in (0, L-L_1) \tag{6}$$

where:

$$[\Phi_1] = \begin{bmatrix} \mathrm{e}^{-ik_1x} & \mathrm{e}^{-ik_2x} & \mathrm{e}^{-ik_1(L_1-x)} & \mathrm{e}^{-ik_2(L_1-x)} \end{bmatrix},$$

$$[\Phi_2] = \begin{bmatrix} \mathrm{e}^{-ik_1(x+L_1)} & \mathrm{e}^{-ik_2(x+L_2)} & \mathrm{e}^{-ik_1[L-(L_1+x)]} & \mathrm{e}^{-ik_2[L-(L_1+x)]} \end{bmatrix},$$

$$\{\alpha_1\} = \mathrm{col}\,[A_1 \quad B_1 \quad C_1 \quad D_1] \text{ and } \{\alpha_2\} = \mathrm{col}\,[A_2 \quad B_2 \quad C_2 \quad D_2]\,,$$

$$[\Psi] = \mathrm{diag}\,[R_1 \quad R_2 \quad -R_1 \quad -R_2]$$

where L_1 denotes the location of the crack, L is the total length of the beam, R_n is the amplitude ratio given by:

$$R_n = \frac{ik_n GAK_1}{GAK_1 k_n^2 - \rho A\omega^2}, \quad K_1 = \left(\frac{0.87 + 1.12\nu}{1+\nu}\right)^2 \text{ for } (n = 1, 2) \quad (7)$$

where ν is Poisson ratio, G is shear modulus, ρ denotes density of the material and ω is a natural frequency.

The wave numbers k_1 and k_2 are roots of characteristic equation in the general form:

$$(GAK_1 EJ)\, k^4 - \left(GAK_1 \rho J K_2 \omega^2 + EJ\rho A\omega^2\right) k^2 + \left(\rho J K_2 \omega^2 - GAK_1\right) \rho A\omega^2 = 0 \quad (8)$$

where: $K_2 = 12K_1/\pi^2$, E denotes Young's modulus and J is the geometrical moment of inertia of the beam cross–section.

The coefficients A_1, B_1, C_1, D_1, A_2, B_2, C_2, and D_2 can be calculated as a function of the nodal spectral displacements using the boundary conditions, as in [5, 6]. The nodal spectral forces can be determined by differentiating the spectral displacements with respect to x. Finally, using relation between the nodal forces and nodal displacements the dynamic stiffness matrix of a cracked rod spectral finite element can be evaluated. Propagation of elastic waves in beams including damping effects is presented in [7].

Delaminated Multilayer Composite Beam Spectral Element

A schematic of a conceptual damage model of a multilayer composite beam is presented in Fig. 5. The beam under consideration is divided into four parts with lengths L_1, $L_2 = L_3$ and L_4. The length of the beam is L, width b, and height h. The delamination is situated between parts 2 and 3. Its length is $L_2 = L_3$.

Displacements and rotations are assumed with Timoshenko beam theory and have the following form:

$$\begin{aligned}
\hat{w}_1(x) &= [\Phi_1] \cdot [\Psi_1] \cdot \{\alpha_1\} \text{ and } \hat{\phi}_1(x) = [\Phi_1] \cdot \{\alpha_1\}, \\
\hat{w}_2(x) &= [\Phi_2] \cdot [\Psi_2] \cdot \{\alpha_2\} \text{ and } \hat{\phi}_2(x) = [\Phi_2] \cdot \{\alpha_2\}, \\
\hat{w}_3(x) &= [\Phi_3] \cdot [\Psi_3] \cdot \{\alpha_3\} \text{ and } \hat{\phi}_3(x) = [\Phi_3] \cdot \{\alpha_3\}, \\
\hat{w}_4(x) &= [\Phi_4] \cdot [\Psi_4] \cdot \{\alpha_4\} \text{ and } \hat{\phi}_4(x) = [\Phi_4] \cdot \{\alpha_4\}
\end{aligned} \quad (9)$$

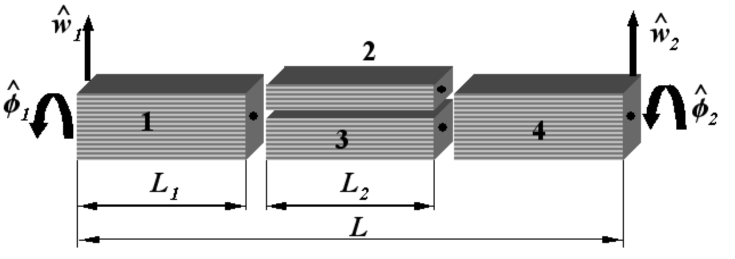

Figure 5. Model of a composite beam with delamination.

where:

$$
\begin{aligned}
[\Phi_1] &= \Big[\mathrm{e}^{-ik_1x} \quad \mathrm{e}^{-ik_2x} \quad \mathrm{e}^{-ik_1(L_1-x)} \quad \mathrm{e}^{-ik_2(L_1-x)}\Big],\\
[\Phi_2] &= \Big[\mathrm{e}^{-ik_1x} \quad \mathrm{e}^{-ik_2x} \quad \mathrm{e}^{-ik_1(L_2-x)} \quad \mathrm{e}^{-ik_2(L_2-x)}\Big],\\
[\Phi_3] &= \Big[\mathrm{e}^{-ik_1x} \quad \mathrm{e}^{-ik_2x} \quad \mathrm{e}^{-ik_1(L_3-x)} \quad \mathrm{e}^{-ik_2(L_3-x)}\Big],\\
[\Phi_4] &= \Big[\mathrm{e}^{-ik_1x} \quad \mathrm{e}^{-ik_2x} \quad \mathrm{e}^{-ik_1(L-x)} \quad \mathrm{e}^{-ik_2(L-x)}\Big],
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
[\Psi_1] &= \mathrm{diag}\Big[R_{1,1} \quad R_{2,1} \quad -R_{1,1} \quad -R_{2,1}\Big],\\
[\Psi_1] &= \mathrm{diag}\Big[R_{1,2} \quad R_{2,2} \quad -R_{1,2} \quad -R_{2,2}\Big],\\
[\Psi_1] &= \mathrm{diag}\Big[R_{1,3} \quad R_{2,3} \quad -R_{1,3} \quad -R_{2,3}\Big],\\
[\Psi_1] &= \mathrm{diag}\Big[R_{1,4} \quad R_{2,4} \quad -R_{1,4} \quad -R_{2,4}\Big],
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
\{\alpha_1\} &= \mathrm{col}\Big[A_1 \quad A_2 \quad A_3 \quad A_4\Big] && \text{for } x \in (0, L_1),\\
\{\alpha_2\} &= \mathrm{col}\Big[A_5 \quad A_6 \quad A_7 \quad A_8] && \text{for } x \in (L_1, L_1 + L_2),\\
\{\alpha_3\} &= \mathrm{col}\Big[A_9 \quad A_{10} \quad A_{11} \quad A_{12}\Big] && \text{for } x \in (L_1, L_1 + L_3),\\
\{\alpha_4\} &= \mathrm{col}\Big[A_{13} \quad A_{14} \quad A_{15} \quad A_{16}\Big] && \text{for } x \in (L_1 + L_3, L)
\end{aligned}
\tag{12}
$$

where $L_{1,2}$ denote the beginning and the end of the delamination area, R_n are the amplitude ratios given by Doyle [1]:

$$
R_{n,j} = \frac{ik_n bA_{66,j} I_{o,j}}{bA_{66,j} I_{o,j} k_n^2 - I_{2,j}\omega^2} \quad \text{for } n = 1, 2, \quad j = 1 \dots 4. \tag{13}
$$

Stiffness coefficient A_{66} describes functions of individual ply properties and orientation, and are integrated over the beam cross–section, I_o and

I_2 are inertia properties of the cross–section. Note that ω is a natural frequency and $i = \sqrt{-1}$. Subscript j denotes the number of the part of the beam element.

The coefficients A_j ($j = 1$ to 16) in Eq. 12 can be calculated as a function of the nodal spectral displacements, taking into account the boundary conditions at the tips of the delamination [8, 9], and the frequency dependent dynamic stiffness matrix, which correlate the nodal spectral forces with the nodal spectral displacements as presented in [10, 11]. The nodal spectral forces can be determined by differentiating the spectral displacements with respect to x.

Spectral Plate Element With a Crack

A model of a spectral plate finite element with a transverse open and non–propagating crack is presented in Fig. 6.

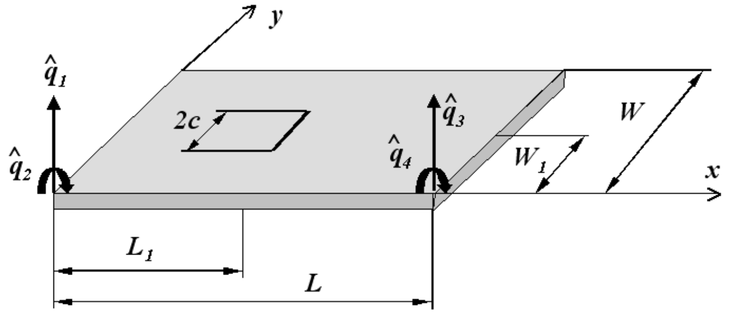

Figure 6. Model of the plate with a transverse open and non–propagating cracks.

The length of the element in the x direction is L, in the y direction is W, and the thickness of the plate is H. The crack is located at a distance of L_1 from the left edge of the plate and has length $2c$.

Nodal spectral displacements (dependent on the wave numbers, consequently on frequencies) are assumed in the following form, for the left (Eq. 14) and right (Eq. 15) part of the plate:

$$\hat{w}_1(x,y) = \sum_{m=1}^{M} \Big(A_{mn} \mathrm{e}^{-ik_1 x} + B_{mn} \mathrm{e}^{-ik_2 x} + C_{mn} \mathrm{e}^{-ik_1 (L_1 - x)} + D_{mn} \mathrm{e}^{-ik_2 (L_1 - x)} \Big) \mathrm{e}^{-i\zeta_m y} \quad \text{for } x \in (0, L_1),\ y \in (0, W), \quad (14)$$

$$\hat{w}_2(x,y)=\sum_{m=1}^{M}\Big(E_{mn}\mathrm{e}^{-ik_1(L_1-x)}+F_{mn}\mathrm{e}^{-ik_2(L_1-x)}+G_{mn}\mathrm{e}^{-ik_1(L-(L_1+x))}$$

$$+H_{mn}\mathrm{e}^{-ik_2(L-(L_1+x))}\Big)\mathrm{e}^{-i\zeta_m y}\quad \text{for } x\in(0,L-L_1),\ y\in(0,W) \quad (15)$$

where: $k_{1,2}$ are the waves numbers calculated as follows:

$$k_1=\sqrt{\beta^2-\xi^2}\quad \text{and}\quad k_2=-i\sqrt{\beta^2+\xi^2} \quad (16)$$

with

$$\xi_m=\frac{2\pi m}{W};\quad \beta^4=\frac{\rho h\omega_n^2-i\eta H\omega_n}{D};\quad D=\frac{EH^3}{12\left(1-\nu^2\right)} \quad (17)$$

where: ρ is the density of the plate material, η is the damping factor, D is the plate stiffness, ν is the Poisson ratio, E denotes Young's modulus and ω_n is a natural frequency. The coefficients A_{mn}, B_{mn}, C_{mn}, D_{mn}, E_{mn}, F_{mn}, G_{mn} and H_{mn} can be calculated as a function of the nodal spectral displacements using the element boundary conditions [12–14].

The nodal spectral forces can be determined by differentiating the spectral displacements with respect to x and y. From the commonly known fact that the dynamic stiffness matrix can relate forces with displacements, the nodal forces can be written as [12–14]:

$$\left\{\hat{F}\right\}=[K]\cdot\{\hat{q}\} \quad (18)$$

where

$$\left\{\hat{F}\right\}=\mathrm{col}\left[\hat{T}\left(0,y\right)\quad \hat{M}\left(0,y\right)\quad \hat{T}\left(L,y\right)\quad \hat{M}\left(L,y\right)\right]$$

$$\{\hat{q}\}=\mathrm{col}\left[\hat{q}_1\quad \hat{q}_2\quad 0\quad \ldots\quad 0\quad \hat{q}_3\quad \hat{q}_4\right]$$

where the square matrix $[K]$ denotes the frequency dependent dynamic stiffness matrix for the spectral plate element with a transverse open and non–propagating crack.

The dimensionless form of the bending flexibility at both sides of the crack can be expressed using formulas investigated by Khadem and Rezee [15], as follows:

$$\theta\left(\frac{\bar{y}}{W}\right)=\frac{6H}{W}\alpha_{bb}\left(\frac{\bar{y}}{W}\right)\cdot F\left(\frac{\bar{y}}{W}\right) \quad (19)$$

where: h is the thickness of the plate, W is the width of the plate, α_{bb} is a function representing the dimensionless bending compliance coefficient and F is a correction function. The function α_{bb} is given in [14]. Propagation of elastic waves in the plate with a transverse open and non–propagating crack is presented in [13, 14].

4. Conclusions

In this study the author has discussed the dynamics of a cracked rod, a cracked Timoshenko beam, a delaminated multilayer composite beam, and cracked plate spectral finite elements. The way of modelling the stiffness loss due to the crack appearance and the excitation force has also been presented. Damage detection is formulated as an optimisation problem which is then solved by using a genetic algorithm.

The results obtained indicate that the current approach is capable of detecting cracks and delaminations of very small size, even in the presence of considerable measurement errors. Only from the differences between the signals measured from undamaged and damaged structures can there be information about the location of the damage. The influence of the growth of the damage for wave propagation in the damaged structure has been shown.

The approach presented is very promising as a fatigue damage detection method. As concluded spectral analysis is very sensitive and allows one to detect damage in its very early state of growth. This fact is extremely important from the practical and economical point of view.

The proposed model can easily be used for detection of damage in more complicated situations, i.e. multiple delaminations located in different places.

The paper is not intended to be a comprehensive survey but merely to present a flavour of recent activity in this important subject.

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