ELASTOPLASTIC MICROSCOPIC BIFURCATION AND POST-BIFURCATION BEHAVIOR OF PERIODIC CELLULAR SOLIDS

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Summary A general framework to analyze microscopic bifurcation and post-bifurcation behavior of elastoplastic, periodic cellular solids is developed on the basis of a two-scale theory of the updated Lagrangian type. We thus derive the eigenmode problem of microscopic bifurcation and the orthogonality to be satisfied by the eigenmodes. By use of the framework, then, bifurcation and post-bifurcation analysis are performed for cell aggregates of an elastoplastic honeycomb subject to in-plane compression. Thus, demonstrating a long-wave eigenmode of microscopic bifurcation under uniaxial compression, it is shown that the eigenmode causes microscopic buckling to localize in a cell row perpendicular to the loading axis. It is also shown that under equi-biaxial compression, the flower-like buckling mode having occurred in a macroscopically stable state changes into an asymmetric, long-wave mode due to the sextuple bifurcation in a macroscopically unstable state, leading to the localization of microscopic buckling in deltaic areas.

INTRODUCTION

When cellular solids are subject to compression, buckling may occur in cell walls and edges. This kind of buckling, which is called microscopic buckling, is of interest from a mechanics point of view because of two features: The first is the complexity of buckling modes, which has been typically observed in hexagonal honeycombs [1, 2]. The second is the macroscopic localization of microscopic buckling. When metallic and polymer honeycombs were transversely compressed, microscopic buckling was likely to localize *in a cell row* and then propagate to the neighboring cell rows, yet *it localized rather broadly under equi-biaxial compression* [2]. Such macroscopic localization may be enhanced by microscopic plastic deformation, since it generally greatly reduces macroscopic stiffness.

In this study, a general framework to analyze microscopic bifurcation and post-bifurcation behavior of elastoplastic, periodic cellular solids is built on the basis of the updated Lagrangian type of two-scale theory developed in [3]. By use of the framework, bifurcation and post-bifurcation analysis are performed for cell aggregates of an elastoplastic honeycomb subject to in-plane compression.

THEORY

We consider an infinite, periodic body $\mathcal{B}$ that has a unit cell Y and is subject to macroscopically uniform stress or strain. A general framework is then established by employing a two-scale theory of the updated Lagrangian type and by taking into account the kY-periodicity of microscopic deformation as well as the multiplicity of microscopic bifurcation. Here, kY indicates a cell aggregate consisting of k unit cells. The framework is generally built without recourse to the symmetry of microscopic bifurcation in contrast to the previous framework [3]. We thus derive the eigenmode problem of microscopic bifurcation, Eq. (1), and the orthogonality to be satisfied by the eingenmodes, Eq. (2):

$$\left\langle l_{ijkl}\phi_{k,l}^{(r)}\delta\dot{u}_{i,j}^{*}\right\rangle = 0\,,\ r = 1, 2, \ldots, m\,, \tag{1}$$

$$\left\langle l_{ijkl}\phi_{k,l}^{(r)}\right\rangle = 0\,, r = 1, 2, \ldots, m\,, \tag{2}$$

where $\langle\ \rangle$ indicates the volume average in kY, l_{ijkl} expresses microscopic stiffness, $\phi_i^{(r)}$ (r = 1, 2, …, m) denote eigenmodes, $\delta\dot{u}_i^{*}$ is any kY-periodic velocity field, $(\ )_{,i}$ represents the differentiation with respect to Cartesian coordinate x_i, and m signifies the degree of multiplicity.

We can show that at the onset of microscopic bifurcation, orthogonality (2) allows the macroscopic increments to be determined independently of the eigenmodes, resulting in a simple procedure of the elastoplastic post-bifurcation analysis based on the notion of comparison solids.

ANALYSIS OF HONEYCOMBS

The theory mentioned above was applied to the in-plane buckling analysis of an elastoplastic honeycomb. The honeycomb was subject to in-plane either uniaxial or equi-biaxial compression. We employed the periodic unit kY that consisted of $M \times N$ subunits, each of which was the aggregate of 2×2 cells illustrated in Fig. 1. This type of periodic units are denoted as $Y_{2M\times 2N}$.

Uniaxial compression

The analysis of uniaxial compression was done by use of the $Y_{2\times 2N}$ type of periodic units. Then, subsequent to a simple bifurcation with no dependence on the periodic cell number 2N, a long-wave bifurcation depending on the periodic cell number 2N occurred in macroscopically unstable states (Fig. 2). The long-wave bifurcation, which occurred earlier with the increase of 2N, was double, i.e., $m = 2$, because of the freedom of a phase shift. The post-bifurcation procedure based on Eq. (2), then, allowed steering into a bifurcated path on which microscopic buckling localized in a cell row perpendicular to the loading axis.

Equi-biaxial compression
The buckling behavior under equi-biaxial compression was analyzed by use of the $Y_{2N\times 2N}$ type of periodic units. Then, as was demonstrated in [3, 4], a triple bifurcation appeared to cause the flower-like buckling mode, which was first found in [2]. Subsequently, a long-wave, sextuple bifurcation occurred soon after initial yielding, if such large periodic units as $Y_{14\times 14}$, $Y_{16\times 16}$, and so on were assumed (Fig. 3). This second bifurcation, which turned out to be asymmetric, induced the localization of microscopic buckling in deltaic areas (Fig. 4 (b)). The localization in such unnarrow regions was likely to occur experimentally [2].

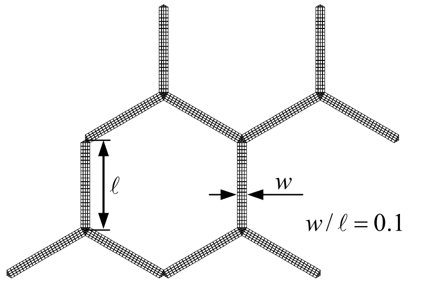


Fig. 1. Subunit of hexagonal honeycomb.

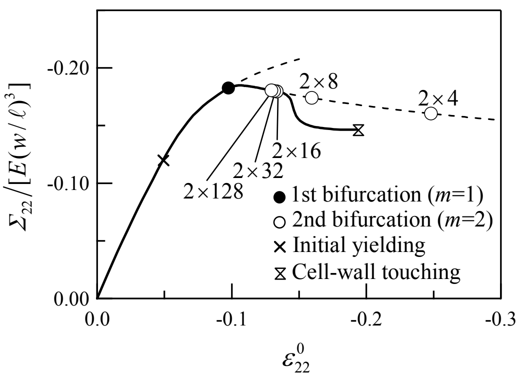


Fig. 2. Macroscopic stress-strain relation under uniaxial compression.

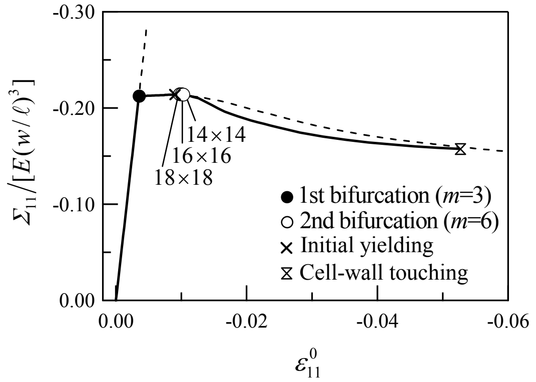
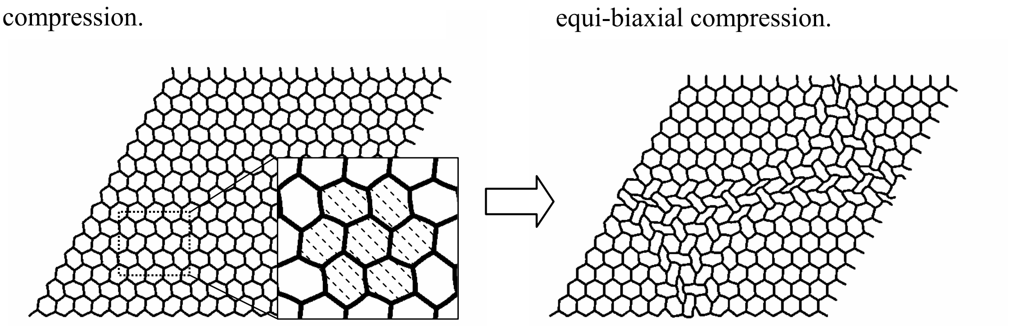


Fig. 3. Macroscopic stress-strain relation under equi-biaxial compression.



(a) $\varepsilon_{11}^{0} = \varepsilon_{22}^{0} = -0.01$ (b) $\varepsilon_{11}^{0} = \varepsilon_{22}^{0} = -0.04$

Fig. 4. Change in deformation of $Y_{16\times 16}$ due to the second bifurcation under equi-biaxial compression.

CONCLUSIONS

On the basis of a two-scale theory of the up-dated Lagrangian type, a general framework was developed to analyze microscopic bifurcation and post-bifurcation behavior of elastoplastic, periodic cellular solids. The framework was applied to the in-plane buckling analysis of honeycombs. We thus had the following findings: Subsequent to the microscopic bifurcation with no dependence on periodic length, the long-wave microscopic bifurcation depending on periodic length occurred in macroscopically unstable states. In the case of equi-biaxial compression, the flower-like buckling mode having occurred in a macroscopically stable state changed into an asymmetric, long-wave mode due to the sextuple bifurcation in a macroscopically unstable state, leading to the localization of microscopic buckling in deltaic areas.

References

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