A viscoplastic model for thermoplastic polymers under uniaxial monotonic and cyclic straining

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ABSTRACT: The thermodynamic approach of the continuous medium and the local state method is used for the study of the mechanical behavior of thermoplastic polymers under uniaxial monotonic and cyclic staining. Viscoplastic constitutive equations are proposed into the restrictive framework of standard generalized materials and the field of the small deformations. A yield criterion is established. A non-linear kinematic hardening rule is proposed to describe cyclic softening that several polymers exhibit under strain controlled cyclic tests.

INTRODUCTION: The increase in using polymers for several industrial applications leads to a strong need of developments of constitutive models. These models are based on either phenomenological or microscopic aspects. The purpose of this paper is to use the general principles of thermodynamics with internal variables for standard generalized materials to predict the thermo-mechanical behavior of polymers. This formalism offers a good framework to introduce viscoplastic constitutive equations. It has widely used for modeling inelastic behavior of polycristalline metals and their alloys. In this case, the assumption of incompressibility and isotropy implies the dependence of the viscoplastic potential only on the second invariant of the stress deviator. This potential is often expressed in the form of a power, exponential or hyperbolic sine yield function. The aim of the present work is to extend the use of this approach to polymeric materials, to propose a generalized yield function, to discuss the various possibilities to express the form of the viscoplastic potential and to discuss the possibilities for the equations governing hardening variables. Assumptions as initial isotropy, homogeneity and normality are checked then adopted.

MATERIALS: Amorphous as well as crystalline thermoplastic polymers are considered in this work: polyethylene terephthalate "PET", polycarbonate "PC", polyoxymethylene "POM" and polyamide 12 "PA12". These polymers are presented in plate shape obtained by extrusion. X-ray measurements and texture analyses were performed on as-received polymers by Siemens diffractometer using Cu-kα x-radiation. The main results corroborated the DSC results show that PET is amorphous on the contrary for other polymers (POM and PA12) crystalline phases have been detected. For texture analyses, POM pole figures of the (110) and (115) reflections show no anisotropy due to its manufacturing. The homogeneity and isotropy of the as received polymers are verified using ultrasonic characterization tests. They were led in immersion and using two transducers of 2.25 MHz.

YIELD FUNCTION: Viscoplastic models formulated into the framework of thermodynamics with internal variables require the definition of a yield function from which is derived the viscoplastic potential. A considerable amount of experimental works dealing with macroscopic yielding of polymers has been done over the last 40 years. It has been established that hydrostatic pressure affects significantly the mechanical behavior of polymers. Therefore, several criteria based on classical plasticity theory involving the hydrostatic component of the stress tensor "I_1" were proposed. Bowden and all [1972] proposed a modified form of the Tresca and the Von Mises criterion. The basic idea they suggested consists to relate linearly the equivalent stress to hydrostatic one. They tested several polymers and concluded that either of both criteria, according to the considered polymer, provides a good description of yielding. This result was highlighted more recently by other authors (Escaig [1997]; Quinson and al. [1997]; Lesser and al. [1997]) who studied yielding of glassy polymers. Raghava and all [1973] have proposed a single yield criterion that is another modification of the Von Mises' one. The results they obtained were in good agreement with prediction. The review of these studies shows that there is no consensus concerning the choice of the most appropriate criterion. To propose a unique yield criterion, we have used experimental results provided from papers mentioned above. For the purpose of this study, experimental data at yield are plotted in the meridional plane (I_1/σ_{st}, J_2/σ_{st}) where σ_{st} is the tensile yield stress. The results that we obtain are:

- Eqn. (1) describes the most suitable criterion that represents the yield behavior.
- The parameter "m" does not depend on strain rate, temperature and molecular weight M_c. Hence, strain rate and temperature effects are introduced in the tensile yield stress σ_{st}.. σ_{sc} is the compressive yield stress.
- For semi-crystalline polymers (PET) a lower von Mises equivalent stress level for pure shear compared to uniaxial tensile at the same hydrostatic pressure.

$$f = \frac{3 \cdot J_2}{\sigma_{st}^2} + (m-1) \cdot \left(\frac{I_1}{\sigma_{st}} \right) - m \quad \text{where} \quad m = \frac{\sigma_{sc}}{\sigma_{st}} \geq 1, \quad I_1 = \mathrm{tr}\left(\underline{\underline{\sigma}}\right) \quad \text{and} \quad J_2 = \frac{1}{2} \cdot tr(\underline{\underline{S}}^2) \tag{1}$$

In order to study the effects of the cristallinity content "X_c" on the polymers yielding and to verify the stress-state behavior dependency, an experimental procedure is developed. Uniaxial as well as biaxial tests are performed at room temperature. The polymers used in the experiments are thermoplastics at the amorphous and semi crystalline state (PA12, PC, POM, amorphous and heat-treated PET). To measure the stress level at which yielding occurs, a conventional value of inelastic strain equal to 0.3% is used. Experimental results show that " X_c " has a negligible influence in the parameter "m". In addition, it appears that the yield function described by Eqn (1) overestimates the shear yielding. To delineate the experimentally observed stress state effects we propose to modify Eqn.(1) by introducing the third invariant of the stress deviator J_3. The expression of the yield function, given by Eqn.**Erreur ! Source du renvoi introuvable.**, where τ_o is the shear yield stress , verifies the convexity conditions. Despite the improvements provided by modifying the yield function, the shear stress at yielding remains overpredicted.

$$f = \frac{3J_2}{\sigma_{st}^2}\Psi(J_2,J_3) + (m-1)\left(1-\frac{\kappa}{8}\right)\frac{I_1}{\sigma_{st}} - (1-\frac{\kappa}{8})m \quad \text{with} \quad \begin{cases} \Psi(J_2,J_3) = \left(1 - \frac{27}{32}\cdot\kappa\cdot\frac{J_3^2}{J_2^3}\right), \; J_3 = \frac{1}{3}\cdot tr\left(\underline{\underline{S}}^3\right) \\ \kappa = \left(1 - \frac{3\cdot r^2}{m}\right) = g(X_c) \;\text{ and }\; r = \frac{\tau_o}{\sigma_{st}} \end{cases} \tag{2}$$

VISCOPLASTIC CONSTITUTIVE EQUATIONS: Strain controlled cyclic tests conducted on several polymers (PA12, PC and PET) in tension-compression show that only non-linear kinematic hardening can be considered to describe hardening that the back stress $\underline{\underline{X}}$ represents. Moreover, before reaching the stabilized cycle cyclic softening is observed leading to consider an evolution of law for $\underline{\underline{X}}$ governed by Eqn. (6) initially proposed for metallic materials. However, as the overall behavior of polymeric materials depends on hydrostatic pressure and considering the assumption of generalized normality, the back stress $\underline{\underline{X}}$ is necessary a non deviatoric tensor. Tow formulations are considered. In the first, recovery effects are neglected so that only the viscoplastic potential is used while for second the dissipation potential is the sum of a viscoplastic potential and a recovery one. The proposed constitutive equations are expressed as follows:

Strain partition : $$\underline{\underline{\varepsilon}} = \underline{\underline{\varepsilon}}^e + \underline{\underline{\varepsilon}}^{in} \quad \text{where} \quad \underline{\underline{\varepsilon}}^e = \underline{\underline{\underline{S}}} : \underline{\underline{\sigma}} \tag{2}$$

Viscoplastic potential : $$\Omega_{vp} = \frac{K}{n+1}\cdot\left\langle\frac{\Gamma}{K}\right\rangle^{n+1} \quad \text{where} \quad \Gamma = F - \frac{\gamma(p)\cdot b(T)}{2}\underline{\underline{\alpha}} : \underline{\underline{\alpha}} \tag{4}$$

$$F = f(\underline{\underline{S}} - \underline{\underline{X}}') + \frac{\gamma(p)}{2\cdot b(T)}\cdot\underline{\underline{X}} : \underline{\underline{X}} \quad , \; \underline{\underline{X}}' : \text{deviator of } \underline{\underline{X}} \tag{5}$$

Kinematic hardening rule: $$\begin{cases} \underline{\underline{X}} = b(T)\cdot\underline{\underline{\alpha}} \;\text{ and }\; \dot{\underline{\underline{\alpha}}} = \dot{\underline{\underline{\varepsilon}}}_{in} - \gamma(p)\cdot\underline{\underline{\alpha}}\cdot\dot{p} \\ \gamma(p) = \gamma_o + \zeta\cdot\left[1 - \exp\left(-\omega\cdot p\right)\right] \end{cases} \tag{6}$$

In order to simulate the behavior of the considered polymers numerical implementation of these equations using Runge-Kutta's method is done. It appears that the proposed model allows a good prediction of cyclic behavior. The introduction of recovery potential improves the prediction of relaxation but has a negligible influence in the other loading conditions.

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