XXI ICTAM, 15-21 August 2004, Warsaw, Poland

# ANALYSIS OF DOUBLE-FREE SURFACE FLOW THROUGH GATES USING ELEMENT-FREE GALERKIN METHOD

Farhang Daneshmand*, Mohammad Javad Kazemzadeh-Parsi**

*Assistant Professor, Mechanical Engineering Department, Shiraz University, Shiraz 71345, Iran

**Graduate Student, Mechanical Engineering Department, Shiraz University, Shiraz 71345, Iran

*Summary* The goal of the present work is to develop a suitable and accurate numerical procedure based on moving least square method and element-free Galerkin method for the computation of free surface profiles, velocity and pressure distributions and flow rate in a two-dimensional gravity fluid flow through a radial gate. The results of the calculations are compared with those obtained from a hydraulic model test.

## PROBLEM STATEMENT AND SOLUTION STRATEGY

In the analysis of free surface flows under gates, computational difficulties arise not only from the non-linear nature of the boundary conditions but also from the fact that the boundary is not known a priori. There are additional difficulties when the problem involves two highly curved unknown free surfaces and arbitrary curved shape boundaries. Fig. 1 shows a typical two-dimensional steady flow from a reservoir through a radial gate as considered in [1]. Geometry of the conduit walls $S_2$ and $S_7$ are given and the far upstream and downstream boundaries of the flow domain, denoted by $S_1$ and $S_4$, are assumed to be normal to the flow direction. The free surface profiles $S_3$ and $S_5$ are not known a priori. In the present study, the flow rate Q is assumed to be unknown and the stagnation level is given as $H_E$, which is the stagnant fluid level above point A. Flow is assumed to be two-dimensional, steady, incompressible, non-viscous and irrotational in which the stream function $\psi$ must obey the Laplace equation

$$\psi_{,xx} + \psi_{,yy} = 0 \tag{1}$$

On the free surfaces $S_5$, $S_3$, two simultaneous conditions should be satisfied; (i) the pressure on the surface must be constant. Using the Bernouli's equation, this condition can be written as

$$\frac{q^2}{2g} + y = H_E \quad \text{or} \quad \frac{\partial \psi}{\partial n} = \sqrt{2g(H_E - y)} \qquad \text{on } S_5,\ S_3 \tag{2}$$

where q is the flow velocity on the free surface, g is the gravity acceleration, y is the free surface elevation, (ii) the velocity normal to the free surface must be zero. This condition can be satisfied through the following procedure based on the algorithm which proposed by Cheng [2] for problems with only one unknown free surface:

(i) Two initial trial free surfaces should be assumed in accordance with the initial trial value of discharge $Q^0$. (ii) The problem is solved for the resulted fixed domain. (iii) The next trial discharge $Q^{(1)}$, can be evaluated from $Q^{k+1} = Q^k + \alpha\left(Q^k - \overline{\psi}^k_{Surf}\right)$ where $\overline{\psi}^k_{Surf}$ is the average of stream function values on the free surface and $\alpha$ is a constant factor (iv) The following equation is used to adjust the location of the lower free surface profile while the upper free surface is taken to be fixed in the first iteration.

$$\Delta y_i = y_i^{k+1} - y_i^k = -\frac{\psi_i^k}{g\left(y_i^k\right)^2}\left[\left(Q^{k+1} - \psi_i^k\right)/\left(1 - \left(\psi_i^k\right)^2/g\left(y_i^k\right)^3\right)\right] \tag{3}$$

(v) The problem is solved again while the new position of the lower free surface is fixed and the upper free surface is allowed to vary. (vi)This iteration process is continued until $\left|Q^{(r+1)} - Q^{(r)}\right| \le \varepsilon$, where $Q^{(r)}$ and $Q^{(r+1)}$ are the discharge values at iteration r and r+1, respectively, and ε is the prescribed accuracy. On the other boundaries, these conditions should be applied, $\psi_{,n}(S_1) = \psi_{,n}(S_4) = \psi(S_2) = 0$, $\psi(S_6) = \psi(S_7) = Q$, where *n* denotes the outward normal direction.

### Element-Free Galerkin Method (EFG)

The element-free Galerkin method (EFG) as proposed in [3] is used in the present study to obtain the following relations for the solution of Laplace equation (Eq. 1):

$$\psi^h(\mathbf{x}) = \sum_{i=1}^{n} \phi_i(\mathbf{x})\hat{\psi}_i \qquad \phi_i(\mathbf{x}) = \sum_{j=1}^{m} P_j(\mathbf{x})\left[\mathbf{A}^{-1}(\mathbf{x})\mathbf{B}(\mathbf{x})\right]_{ji} \tag{4}$$

$$\mathbf{A}(\mathbf{x}) = \sum_{i=1}^{n} w_i(\mathbf{x})\mathbf{P}(\mathbf{x}_i)\mathbf{P}^T(\mathbf{x}_i) \qquad \mathbf{B}(\mathbf{x}) = \left[w_1(\mathbf{x})\mathbf{P}(\mathbf{x}_1), w_2(\mathbf{x})\mathbf{P}(\mathbf{x}_2), \ldots, w_n(\mathbf{x})\mathbf{P}(\mathbf{x}_n)\right] \tag{5}$$

where $\mathbf{p}^T(\mathbf{x}) = \left[p_1(\mathbf{x}), p_2(\mathbf{x}), \ldots p_m(\mathbf{x})\right]$ contains *m* basis function, $w_i(\mathbf{x})$ is the weight function associated with the node *i* and evaluated at point x. In this paper the cubic spline weight function is used,

$$W = \begin{cases} (2 - 12s^2 + 12s^3)/3 & s \le 1/2 \\ (4 - 12s + 12s^2 - 4s^3)/3 & 1/2 \le s \le 1 \\ 0 & s > 1 \end{cases} \tag{6}$$

in which $s = r/R_m$ is a normalized radius and $R_m$ is the support size. Finally,

XXI ICTAM, 15-21 August 2004, Warsaw, Poland

$$(\mathbf{K}-\mathbf{G})\Psi=\mathbf{q} \qquad (7)$$

$$\mathrm{K}_{ij}=\int_{\Omega}\left(\phi_{i,x}\,\phi_{j,x}+\phi_{i,y}\,\phi_{j,y}\right)d\Omega\,,\;\; \mathrm{G}_{ij}=\int_{\Gamma_u}\phi_i\left(\phi_{j,x}n_x+\phi_{j,y}n_y\right)d\Gamma\,,\;\; q_i=\int_{\Gamma_q}\phi_i\,\bar{q}\,d\Gamma \qquad (8)$$

where $\phi_i(\mathbf{x})$ is the shape function of $i^{th}$ node at point **x**, and $\Gamma_u$ and $\Gamma_q$ are parts of boundary with essential and natural boundary conditions, respectively. As it is seen from the above equations, the essential boundary condition is applied using the method proposed in [4] through the matrix $G_{ij}$.

## RESULTS

The geometric details of the radial gate in are given in Fig. 1 ($H_o$=3.48 m, gate opening = 30%). To ensure the validity of the results obtained for this example, a hydraulic model test based on Froude law of similarity is also constructed [5]. The pressure distribution in the liner is shown in Fig. 2 and is compared with those obtained from the experiment. Fig. 3 shows the pressure distribution on the radial gate obtained from the numerical and experimental analyses. The shapes of upper and lower free surfaces are plotted in Fig. 4. The computed discharge is 0.470 $m^3$/s.m that is in good agreement with the experimental result, 0.485 $m^3$/s.m.

## CONCLUSION

This paper demonstrates that the element-free Galerkin method can be successfully used in the free surface flow through the radial gates. It is found that the present procedure can easily handle the problems involving curved shape solid boundaries and is also quite efficient in locating two free surface profiles. It converges rapidly and the results obtained are in good agreement with the experiment.

## ACKNOWLEDGMENT

The assistance of Sadid-Tadbir Co. is gratefully acknowledged.

## References

[1] F. Daneshmand, S. K. Sharan and M. H. Kadivar: Finite Element Analysis of Double-Free-Surface Flow through Slit in Dam. Journal of Engineering Mechanics, ASCE, Vol. 126, No. 5, 515-523, 2000.
[2] A. H-D. Cheng, J. A. Liggett and P. L-F. Liu: Boundary calculations of Sluice and spillway flows. Journal of Hydraulic Engineering, ASCE 107, HY 10, 1163-1178, 1981.
[3] T. Belytschko, Y. Krongauz, D. Organ, M. Flemming and P. Krysl: Meshless Methods: An Overview and Recent Developments. Computer Methods in Applied Mechanics and Engineering, 139, 3-47, 1996.
[4] F. Daneshamand and M. J. Kazemzadeh Parsi: A Meshless Method for Free Surface Flow Through Sluice Gates. 6th International Conference on Hydro-informatics, Singapore, June 2004 (Submitted).
[5] Hydraulic model test of the Orifice Spillway for Karun III Dam, Report No. KRM-R-002, Sadid-Tadbir Co., Tehran, Iran, 1997.

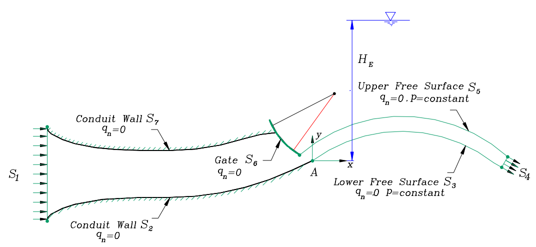


Fig. 1 Flow under the radial gate

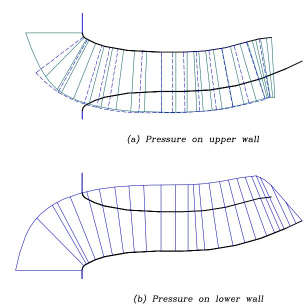


Fig. 2 Pressure in the liner

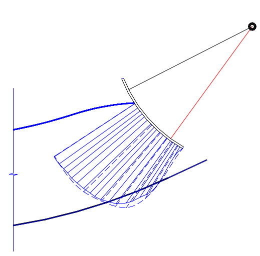

Fig. 3 Pressure on the radial gate

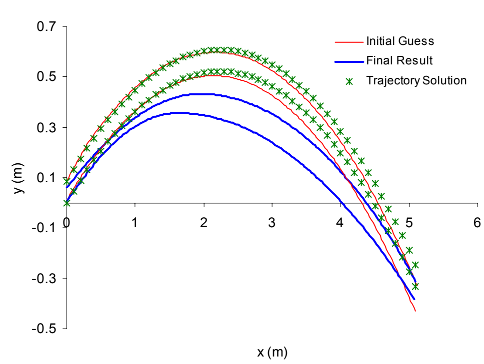


Fig. 4 Free surface Profiles