ADVANCED THERMO-VISCO-PLASTIC CONSTITUTIVE RELATIONS FOR DIRECT APPLICATIONS IN NUMERICAL ANALYSES

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Summary In this contribution thermo-visco-plastic constitutive relations are proposed and analyzed. This phenomenological approach is partly based on theory of dislocations and processes of thermal activation. The main advantages of such constitutive relation are reduction of number of material constants in comparison with the complete approach based on materials science (theory of dislocations). This constitutive relation has been used so far many times in finite element codes to simulate dynamic processes of impact loading. An original algorithm using J_2 plasticity theory proposed by Zaera et al. [4] has been recently applied in this study.

CONSTITUTIVE RELATIONS

The original constitutive relation proposed in this paper is based on the formalism proposed by Klepaczko [1] which applies theory of dislocations. In the original approach, the evolution of microstructure is taken into account by internal variables S_j .

$$\sigma = g\left[\varepsilon, \dot{\varepsilon}, T, S_j(\varepsilon, \dot{\varepsilon}, T)\right] \tag{1}$$

However, this complete model is complicated to be implemented into a Finite Element Code due to the complexity of each equation coupled with the great number of physical constants in comparison with phenomenological approach. To reduce the number of constant and to facilitate the utilization in a FE codes, a semi phenomenological approach has been proposed by Rusinek and Klepaczko [2] to simplify modelling by physical intuition. Thus, the plastic flow stress is defined as follow :

$$\sigma(\varepsilon, \dot{\varepsilon}, T) = \frac{E(T)}{E_0}\left[\sigma_\mu(\varepsilon, \dot{\varepsilon}, T) + \sigma^*(\dot{\varepsilon}, T)\right] \quad \text{with} \quad E(T) = E_0\left\{1 - \frac{T}{T_m}\exp\left[\theta^*\left(1 - \frac{T_m}{T}\right)\right]\right\} \tag{2}$$

where $E(T)$ is the temperature-dependent Young's modulus with the characteristic homologous temperature θ^*, E_0 is the Young's modulus at $T = 0\,K$, T_m is the melting temperature.

The explicit form proposed to define the two stress components are inspired by the physical approach [1] and the theory of thermal activation [3]. Thus

$$\sigma_\mu = B(\dot{\varepsilon}, T)(\varepsilon_0 + \varepsilon)^{n(\dot{\varepsilon}, T)}, \quad \sigma^* = \sigma_0^*\left[1 - D_1\left(\frac{T}{T_m}\right)\log\left(\frac{\dot{\varepsilon}_{max}}{\dot{\varepsilon}}\right)\right]^{m^*} \quad \text{with} \quad \dot{\varepsilon} \neq 0 \quad \text{and } \sigma^* \geq 0 \tag{3}$$

Best way to define correctly the effect of temperature is to assume that temperature effect on plastic deformation is taken into account by two expressions.

$$n(\dot{\varepsilon}, T) = n_0\left[1 - D_2\left(\frac{T}{T_m}\right)\log\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_{min}}\right)\right] \quad \text{with} \quad n \geq 0 \qquad B(\dot{\varepsilon}, T) = B_0\left[\frac{T}{T_m}\log\left(\frac{\dot{\varepsilon}_{max}}{\dot{\varepsilon}}\right)\right]^{-\nu} \tag{4}$$

where n_0 is the strain hardening exponent at $T = 0\,K$, D_2 is a constant, $\dot{\varepsilon}_{min}$ is the minimal strain rate assumed in the model, B_0 is a constant, $\dot{\varepsilon}_{max}$ is the maximal strain rate assumed in the model and ν is the temperature sensitivity.

Reduced number of constant is eight and it allows to approximate the same tendencies as predicted by the physical approach. A complete optimization algorithm has been written to determine the unique optimal set of constants for each material [2]. In order to apply the set of constitutive relations a integration scheme must be applied. Such original integration scheme has been proposed by Zaera et al. [4].

IMPLICIT INTEGRATION ALGORITHM

The yield stress σ_y is defined by Eq. 2, $(\sigma_y = \sigma)$, which takes into account strain hardening, strain rate sensitivity and temperature effect. Thus in general loading conditions, the yield surface is defined as follows:

$$f\left(\sigma_{ij}, \bar{\varepsilon}^p, \dot{\bar{\varepsilon}}^p, T\right) = \bar{\sigma} - \sigma_y\left(\bar{\varepsilon}^p, \dot{\bar{\varepsilon}}^p, T\right) = 0 \quad \text{with} \quad \bar{\sigma} = \sqrt{\frac{3}{2} s_{ij} : s_{ij}} \tag{5}$$

where $\bar{\sigma}$ is the equivalent stress, $\bar{\varepsilon}^p$ is the equivalent plastic strain, $\dot{\bar{\varepsilon}}^p$ is the equivalent plastic strain rate and s_{ij} is the deviatoric part of the stress tensor σ_{ij}

In this approach, the total strain tensor ε_{ij} is a sum of the elastic strain tensor ε_{ij}^e, the plastic strain tensor ε_{ij}^p and the thermal strain ε_{ij}^θ . To define the plastic flow, the normality rule is used allowing to relate plastic strain rate $\dot{\varepsilon}_{ij}^p$ to the stress. The last is defined as follows:

$$\dot{\varepsilon}_{ij}^p = \dot{\lambda} \frac{\partial f}{\partial \sigma_{ij}} \quad \text{with} \quad \dot{\lambda} > 0\, ; \quad \dot{\bar{\varepsilon}}^p = \sqrt{\frac{2}{3} \dot{\varepsilon}_{ij}^p : \dot{\varepsilon}_{ij}^p} \quad \text{and} \quad \bar{\varepsilon}^p = \int \dot{\bar{\varepsilon}}^p \, dt \tag{6}$$

where $\dot{\lambda}$ is the plastic multiplier. In classical overstress models [5], the consistency condition is not used and excursions of stress outside the yield surface are allowed. Several authors have proposed the so called “consistency viscoplasticity models” to include rate effects in the consistency condition, Eq. 7. Following the work of Winnicki et al. [6], who used the approach $\dot{\bar{\varepsilon}}^p = \Delta\bar{\varepsilon}^p / \Delta t$ to include viscoplastic effects in the consistency condition for the yield surface, the present algorithm is developed for J_2 plasticity including thermal effects

$$\dot{f}\left(\sigma_{ij}, \bar{\varepsilon}^p, \dot{\bar{\varepsilon}}^p, T\right) = \frac{\partial f}{\partial \sigma_{ij}} \dot{\sigma}_{ij} + \frac{\partial f}{\partial \bar{\varepsilon}} \dot{\bar{\varepsilon}}^p + \frac{\partial f}{\partial \dot{\bar{\varepsilon}}^p} \ddot{\bar{\varepsilon}}^p + \frac{\partial f}{\partial T} \dot{T} = 0 \quad \text{with} \quad \dot{T} = \frac{\beta}{\rho C_p} \sigma_{ij} : \dot{\varepsilon}_{ij}^p \tag{7}$$

where $\ddot{\bar{\varepsilon}}^p$ is the equivalent plastic acceleration, ρ is the density of material, β is the Quinney-Taylor coefficient and C_p is the specific heat at constant pressure assumed constant during numerical simulations.

The linearization of the consistency equation, Eq. (7), leads to Eq. (8) where k is an iterative index [4].

$$f_{k+1} \approx f_k + \left.\frac{\partial f}{\partial \sigma_{ij}}\right|_k : \left(-\delta\Delta\bar{\varepsilon}_k^p 2G \left.\frac{\partial f}{\partial \sigma_{ij}}\right|_k \right) + \left.\frac{\partial f}{\partial \bar{\varepsilon}^p}\right|_k \delta\Delta\bar{\varepsilon}_k^p + \left.\frac{\partial f}{\partial \dot{\bar{\varepsilon}}^p}\right|_k \frac{\delta\Delta\bar{\varepsilon}_k^p}{\Delta t} + \left.\frac{\partial f}{\partial T}\right|_k \frac{\beta}{\rho C_p} \left(\delta\Delta\bar{\varepsilon}_k^p \bar{\sigma}_{n+1}^{trial} - 6G\delta\Delta\bar{\varepsilon}_k^p \delta\Delta\bar{\varepsilon}_k^p \right) = 0 \tag{8}$$

This leads to an iterative Newton-Raphson scheme, which is stopped when the residual f_{k+1} is small enough. One advantage of this algorithm is its similarity to those previously employed in the rate independent and temperature independent plasticity. The complete characterization of the algorithm is given in [4]. Concerning the unloading conditions, the Hooke’s law is programmed causing an instantaneous unloading.

CONCLUSIONS

The combination of the thermoviscoplastic model and a new integration scheme taking into account the strain hardening, strain rate sensitivity and temperature effects, allows to simulate and study variety of processes of dynamic loading and impact. Up to now, the constitutive relations have been used without the new algorithm in [4] to analyze impact behavior for several kinds of sheet steel as mild steel or DP steel. The numerical analyses were also applied to study the problem of elastic-plastic wave propagation in sheet metals[7].

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