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Three-Dimensional Transmission in Plane Layered Elastic Composites

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Extended Summary

We already have a complete theory of three-dimensional elastic images in the case of two linearly elastic isotropic media (Aderogba, 1977) separated by a flat surface. Whatever be the distribution of elastic singularities in the first medium, the elastic field induced in the two media can instantly be calculated by routine differentiation and integration, without going through the rigour of solving the equilibrium equations and satisfying the necessary interface continuity conditions.

Specifically, if u_i is the displacement field induced by any distribution of singularities in an unbounded isotropic elastic solid, then the specification of the same distribution of singularities in an elastic upper half-space z > 0 which is perfectly bonded to a dissimilar elastic lower half-space z < 0 yields the displacement fields

$$u_i^{(1)} = u_i(x, y, z) + L_{ij}^{(1)} \quad u_j(x, y, -z)$$

for z > 0, i, j = 1, 2, 3, and

$$u_i^{(2)} = L_{ij}^{(2)} \quad u_j(x, y, z)$$

for $z < 0$, where $L_{ij}^{(k)}$ are suitably defined integro-differential operators which contain combinations of the elastic moduli of the two isotropic half- spaces. The effect of a single flat surface of material discontinuity thus appears like an ordinary translation and rotation of rectangular Cartesian coordinate system.

It therefore seems logical to predict that a similar complete theory of elastic images is possible in the generalized case of three media separated by two parallel plane surfaces. This prediction is actually inspired by Maxwell's (1873) magnificient compact solution of the corresponding electrostatic problem of electrical conduction through a thick plate separating two other dissimilar dielectric media. We may also refer to another inspiration, again by Maxwell (1873) who provides an instructive image treatment to the problem of a point charge arbitrarily placed between two conducting grounded non-intersecting spherical surfaces.

The present investigation is therefore concerned with the determination and characterization of the three-dimensional image system produce by an arbitrary elastic singularity which is operative in or near a thick layer which separates two other dissimilar isotropic semi-infinite solids. This determination naturally hinges on a systematic stepwise application of the aforementioned complete theory of images in the case of two media, resulting in an infinite series representation which does not involve the introduction of any special function. From the series, general interesting asymptotic values can be extracted.

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