XXI ICTAM, 15–21 August 2004, Warsaw, Poland

TRANSIENT DYNAMIC CRACK ANALYSIS IN FGMS UNDER IMPACT LOADING

Chuanzeng Zhang*, Jan Sladek**, Vladimir Sladek**
*Department of Civil Engineering, University of Applied Sciences Zittau/Görlitz, D-02763 Zittau, Germany
**Institute of Construction and Architecture, Slovak Academy of Sciences, 84503 Bratislava, Slovakia

Summary Transient elastodynamic analysis of a two-dimensional (2-D) in-plane crack in functionally graded materials (FGMs) is performed by a time-domain boundary integral equation method (BIEM). An exponential law is applied to describe the material gradients of the FGMs. The crack is subjected to an impact crack-face loading. Special attention is devoted to explore the effects of the material gradients and the crack orientation on the dynamic stress intensity factors (DSIFs) and their overshoot over static values.

PROBLEM STATEMENT AND BOUNDARY INTEGRAL EQUATIONS

Transient elastodynamic crack analysis in FGMs is of particular interest to fracture mechanics and ultrasonic quantitative non-destructive evaluation of FGMs, which received in last years more and more attention in material sciences and engineering applications due to their superior thermal and mechanical properties. Due to the mathematical complexity arising in such an analysis, most of the previous works on crack analysis in FGMs have been limited to very special crack orientation and loading conditions. In this paper, we present a transient elastodynamic crack analysis in unidirectional or bidirectional FGMs subjected to an impact crack-face loading.

We consider an infinite, isotropic, continuously non-homogeneous, and linear elastic solid containing a finite crack of length $2a$ as shown in Fig. 1.

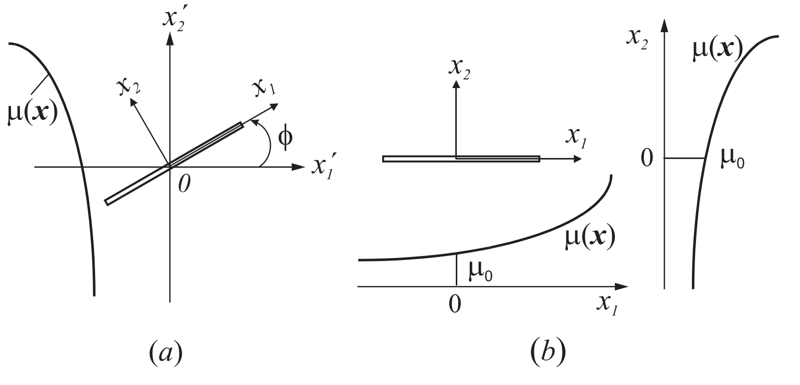

Figure 1. A finite crack in FGMs with (a) unidirectional gradation and (b) bidirectional gradation

The crack is subjected to an impact crack-face loading, and the deformation of the solid is in plane strain or plane stress. In the absence of body forces, the cracked FGMs satisfy the equations of motion

$$\sigma_{\alpha\beta,\beta} = \rho(\mathbf{x})\ddot{u}_\alpha \, , \tag{1}$$

the Hooke's law

$$\sigma_{\alpha\beta} = \mu(\mathbf{x}) E^0_{\alpha\beta\delta\gamma} u_{\delta,\gamma} \, , \tag{2}$$

the initial conditions

$$u_\alpha(\mathbf{x},t) = \dot{u}_\alpha(\mathbf{x},t) = 0 \, , \qquad \text{for} \quad t = 0 \, , \tag{3}$$

and the boundary conditions on the crack-faces

$$f_\alpha(\mathbf{x},t) = \sigma_{\alpha\beta}(\mathbf{x},t) n_\beta(\mathbf{x}) = f^*_\alpha(\mathbf{x},t) \, , \qquad \mathbf{x} \in \Gamma_c \, . \tag{4}$$

In Eqs. (1)-(4), $\rho(\mathbf{x})$ is the mass density, $\mu(\mathbf{x})$ is the shear modulus, $\Gamma_c = \Gamma_c^+ + \Gamma_c^-$ are the crack-faces, n_β is the unit normal vector, $f^*_\alpha(\mathbf{x},t)$ is a pre-scribed crack-face loading, and

$$E^0_{\alpha\beta\delta\gamma} = \frac{3-\kappa}{\kappa-1}\delta_{\alpha\beta}\delta_{\delta\gamma} + \delta_{\alpha\delta}\delta_{\beta\gamma} + \delta_{\alpha\gamma}\delta_{\beta\delta} \, . \tag{5}$$

Here, $\kappa = 3 - 4\nu$ for plane strain, $\kappa = (3-\nu)/(1+\nu)$ for plane stress, ν is Poisson's ratio which is assumed to be constant in this analysis, and $\delta_{\alpha\beta}$ is the Dirac-delta. The spatial variations of the shear modulus and the mass density are described by the following exponential laws

$$\mu(\mathbf{x}) = \mu_0 e^{\alpha x_1 + \beta x_2} \, ; \qquad \rho(\mathbf{x}) = \rho_0 e^{\alpha x_1 + \beta x_2} \, , \tag{6}$$

where α and β are gradient parameters of the FGMs. The exponential law (6) enables us to describe crack problems in both unidirectional and bidirectional FGMs as shown in Fig. 1.

The initial-boundary value problem governed by Eqs. (1)-(4) can be formulated as a set of time-domain traction BIEs as

$$n_\beta(\mathbf{x}) \int\limits_{\Gamma_c^+} T^G_{\gamma\alpha\beta}(\mathbf{x},\mathbf{y};t,\tau) * \Delta u_\gamma(\mathbf{y},\tau) ds = f^*_\alpha(\mathbf{x},t) \,, \qquad \mathbf{x} \in \Gamma_c^+ \,, \tag{7}$$

where $T^G_{\gamma\alpha\beta}$ are the time-dependent traction Green's functions, Δu_α represent the unknown crack-opening-displacements (CODs), and an asterisk denotes Riemann convolution. Note here that the time-domain traction BIEs (7) are hypersingular. The hypersingular integral in (7) has to be understood in the sense of Hadamard finite-part integral.

NUMERICAL SOLUTION PROCEDURE

A time-stepping scheme is developed for solving the hypersingular time-domain BIEs (7). The scheme uses the convolution quadrature formula of Lubich [1] for computing the temporal convolution integral and a Galerkin-method for the spatial approximation of the unknown CODs. The unknown CODs are approximated by

$$\Delta u_\gamma(y_1,\tau) = \sqrt{a^2 - y_1^2} \sum_{k=1}^{K} c_{\gamma;k}(\tau) U_{k-1}\left(\frac{y_1}{a}\right) \,, \tag{8}$$

where K is the number of series, $c_{\gamma;k}(\tau)$ are the unknown time-dependent expansion coefficients, and $U_{k-1}(y_1/a)$ are the Chebyshev-polynomials of second kind. Substituting Eq. (8) into Eq. (7), multiplying both sides by $\sqrt{a^2 - x_1^2} U_{l-1}(x_1/a)$, integrating them with respect to x_1 from $-a$ to $+a$, and applying the convolution quadrature formula of Lubich [1] lead to a system of linear algebraic equations for the expansion coefficients

$$\sum_{j=0}^{n-1} \mathbf{A}^{n-j} \mathbf{d}^j = \mathbf{t}^n \,, \qquad n = 1, 2, ..., N \,, \tag{9}$$

where $\mathbf{d}^j = \{\mathbf{c}_1^j, \mathbf{c}_2^j\}^T$ and $\mathbf{t}^n = \{\mathbf{f}_1^n, \mathbf{f}_2^n\}^T$, with $\mathbf{c}_\gamma^j = \{c^j_{\gamma;k}\}$ and $\mathbf{f}_\gamma^n = \{f^n_{\gamma;l}\}$. Here, the time variable t is divided into N equal time-steps Δt and the upper indices stand for the time-steps. The system matrix and the right-hand side of Eq. (9) can be obtained by using

$$\mathbf{A}^n = \frac{r^{-n}}{N} \sum_{m=0}^{N-1} \hat{\mathbf{A}}(p_m) e^{-2\pi i \cdot nm/N} \,, \tag{10}$$

$$f^n_{\alpha;l} = (-1)^{l+1} \int\limits_{-a}^{+a} f^*_\alpha(x_1, n\Delta t) \sqrt{a^2 - x_1^2} U_{l-1}\left(\frac{x_1}{a}\right) dx_1 \,. \tag{11}$$

In Eq. (10), $p_m = \delta(\zeta_m)/\Delta t$, $\delta(\zeta_m) = \sum_{j=1}^{2}(1-\zeta_m)^j/j$, $\zeta_m = re^{2\pi i \cdot m/N}$, and $r^N = \sqrt{\epsilon}$ with ϵ being the numerical error arising in the computation of the Laplace-domain system matrix $\hat{\mathbf{A}}(p_m)$. An essential feature of the present time-domain method is that it uses the Laplace-domain instead of the time-domain Green's functions, which are yet not available in literature for FGMs. An explicit expression of the time-domain Green's functions $T^G_{\gamma\alpha\beta}$ is not required in the present method. The Laplace-domain Green's functions $\hat{T}^G_{\gamma\alpha\beta}$ are expressed as Fourier-integrals. The computation of Eq. (10) can be performed very efficiently by using the Fast-Fourier-Transform (FFT). The system of linear algebraic equations (9) can be solved time-step by time-step to obtain the expansion coefficients $c^j_{\gamma;k}$. The DSIFs can be computed immediately in a simple manner.

NUMERICAL RESULTS AND CONCLUSIONS

Numerical results obtained by the present time-domain BIEM show that the method is highly accurate, efficient and stable. Several numerical examples will be presented, to analyze the effects of the material gradients and the crack orientation with respect to the material gradation on the DSIFs and their dynamic overshoot over the corresponding static values.

To the authors knowledge, the results presented in this paper are completely new and they cannot be found elsewhere in literature. The corresponding anti-plane crack problem has been investigated in [2], while a 2-D elastostatic crack analysis in FGMs has been performed in [3] by the authors .

References

[1] Lubich, C.: Convolution Quadrature and Discretized Operational Calculus. I. *Numerische Mathematik* **52**: 129–145, 1988.

[2] Zhang, Ch., Sladek, J., Sladek, V.: Effects of Material Gradients on Transient Dynamic Mode-III Stress Intensity Factors. *Int. J. Solids Struct.* **40**:5251–5270, 2003.

[3] Zhang, Ch., Sladek, J., Sladek, V.: Numerical Analysis of Cracked Functionally Graded Materials. *Key Eng. Mater.* **251-252**:463-471, 2003.