VISCOUS AND VISCOELASTIC POTENTIAL FLOW

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Abstract

Recent results will be reviewed and new results presented which establish that in all cases in which potential flow satisfies the equations of motion for viscous (or viscoelastic) fluids, it is neither necessary nor useful to put the viscosity to zero. Stated more severely these results suggest that the inviscid part of potential flow theory may be deleted.

Extended Summary

It is well known that the Navier-Stokes equations are satisfied by potential flow; the viscous term is identically zero when the vorticity is zero but the viscous stresses are not zero (Joseph & Liao 1994). It is not possible to satisfy the no-slip condition at a solid boundary or the continuity of the tangential component of velocity and shear stress at a fluid-.fluid boundary when the velocity is given by a potential. The viscous stresses enter into the viscous potential flow analysis of free surface problems through the normal stress balance at the interface. Viscous potential flow is an approximation to viscous flow which neglects the effects of vorticity and friction at the boundary. Two questions are immediately suggested by such an approximation. Is viscous potential flow a better approximation than inviscid potential flow? (1) Under what circumstances is viscous potential flow a better approximation than inviscid potential flow? (2) Under what circumstances is viscous potential flow a good approximation to viscous flow? The answer to the first question is yes; general answers to the second question are not yet known; the subject is new and under development.

Up to now, the utility of the theory of potential flow of a viscous fluid has been suggested by the comparison of potential and viscous flow solutions and experiments and some partial results from flow fundamentals. The examples show that when considering potential flow, it is never necessary nor useful to put the viscosity to zero.

The entry “potential flow” in search on Google yields 8,770,000 hits. Of these, the number of academic studies of potential flows of viscous fluids is perhaps less than twenty. These studies may be divided into applications using the dissipation method and to the direct application to interface problems, especially to stability problems of the type considered here. The dissipation method has been applied to the calculation of drag on gas bubbles of different shapes rising in viscous liquids and to computation of the rates of decay of waves on gas-liquid surfaces discussed by Lamb 1932 (see Joseph & Wang 2003 for a recent review together with new results). Harper 1972 argued that the dissipation method works well for the drag on a spherical gas bubble at large Re because, unlike the solid sphere, an increasing fraction of the dissipation occurs in the irrotational bulk liquid relative to the dissipation in the vorticity layer at the boundary. The boundary layer at a solid surface resolves a discontinuity of velocity. At a gas surface it resolves a discontinuity in the vorticity. Harper’s idea is plausible though details have not been given. On the other hand, stability limits for capillary instability of jets of viscous liquids into viscous liquids using the potential flow approximation is very close to exact results for moderately large Reynolds numbers (Funada & Joseph 2002). In other words, there are cases for which viscous potential flow works well at liquid-liquid surfaces where Harper’s 1972 arguments do not apply; it is a surprise and not well understood.

Uncompensated irrotational shear stresses arise at stress free surfaces when the flow is irrotational, Joseph & Wang 2003 argued that irrotational flow of a viscous fluid is a good approximation when the uncompensated irrotational shear stress is zero or small. The Rayleigh-Plesset bubble executes purely radial motions in a viscous liquid; the motion of the liquid is exactly described by irrotational flow; it depends on viscosity through the viscous normal stress at the gas-liquid surface. There are no uncompensated shear stresses there and the solution is exact. The rise of a spherical cap bubble is a case where the uncompensated shear stress near the stagnation point is small. It is very well approximated by the potential flow of a viscous fluid (Joseph 2003). Joseph, Belanger & Beavers 1999 constructed a viscous potential flow analysis of the Rayleigh-Taylor instability which gives results which are almost indistinguishable from those obtained from the exact fully viscous analysis. The basic flow whose stability is studied is free of shear stresses.

Other stability studies using potential flow of a viscous fluid are not uniformly excellent approximations of exact results, but the errors are small for some parameter regions, with large errors less than about 40% in the worst cases. In all cases, viscous potential flow gives a much better approximation to exact results and experiments, than inviscid potential flow.

Funada & Joseph 2002 constructed a viscous potential flow analysis of capillary instability of a liquid cylinder which was in excellent agreement with the exact fully viscous analysis. Funada & Joseph 2001 constructed a viscous potential flow analysis of Kelvin-Helmholtz instability in a channel. A fully viscous flow analysis is not available because the basic flow which postulates two uniform streams with different velocities is incompatible with the requirement that the shear stress and tangential component of velocity should be continuous and the no-slip conditions at the channel wall. The analysis of Funada & Joseph 2001 is in much better agreement with experiments reviewed by Mata, Pereyra, Trallero & Joseph 2002 than with other theories which account for the shear of the gas using different empirical correlations.

The excellent and well known book on stability theory by Drazin & Reid 1981 starts with an analysis of Rayleigh-Taylor, Kelvin-Helmholtz and capillary instability of an inviscid fluid. The potential flow analyses are not more difficult, but have a much richer content when the viscous contribution to the normal stress is not put to zero.

Irrotational flows of viscous fluids are not good approximations for flows with distributed vorticity. The diffusion of vorticity from the boundary is important. For stability problems, the time for instability could be smaller than a typical time of diffusion. However, there are flows such as rising gas bubbles in which the diffusion of vorticity is not an important factor.

Here, we study the potential flow of a second order fluid over a sphere or an ellipse. The normal stress at the surface of the body is calculated and has contributions from the inertia, viscous and viscoelastic effects. We investigate the effects of different Reynolds numbers and body sizes on the normal stress; for the ellipse, various angles of attack and aspect ratios are also studied. The effect of the viscoelastic terms is opposite to that of inertia; the normal stress at a point of stagnation can change from compression to tension. This causes long bodies to turn into the stream and causes spherical bodies to chain. For a rising gas bubble, the effect of the viscoelastic and viscous terms in the normal stress is to extend the rear end so that it tends to the cusped trailing edge observed in experiments.

References

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