TREESPH SIMULATIONS OF CHOKED FLOW SYSTEMS USING SMOOTHED PARTICLE HYDRODYNAMICS

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Summary We present exploratory two-dimensional calculations of the flow of a viscous, single-phase fluid through a wellhead choke of real dimensions using the method of Smoothed Particle Hydrodynamics coupled with a simple isothermal equation of state for description of the flow.

The flow of gas-liquid mixtures through restrictors, like flow control valves and chokes, in pipeline systems is of great practical interest in many applied branches of engineering. In the oil industry, wellhead chokes are installed to control flow rates and protect the surface equipment from unusual pressure fluctuations. Due to its practical significance, single-phase and two-phase flows through chokes have been the subject of numerous investigations in the past 40 years. However, the complexity of the problem has limited the investigation mostly to the development of empirical correlations based on experimental measurements and theoretical studies based on simplified treatments. In general, when a flowing mixture crosses a choke its velocity increases and its pressure drops. The empirical correlations aimed at predicting the dependence of the pressure drop on velocity through the choke are usually valid over the range where experimental data are available but may fail when extrapolated to new conditions. With the exception of a very few instances, such simulations are practically inexistent in the literature even for the case of single phase flows.

In this paper, we search for a way of solving the Navier-Stokes equations in order to simulate the flow of a single-phase, viscous fluid through a wellhead choke using a modified version of the TREESPH method originally introduced by Hernquist & Katz [1]. In particular, a Smoothed Particle Hydrodynamics (SPH) formulation is used which has been shown to produce accurate results for both compressible flows at high and moderate Reynolds numbers and incompressible flows at low Reynolds numbers without the need of special modifications. Unlike other SPH-based schemes for treating viscous flows, the present method strictly relies on symmetrized SPH representations for the equations of motion and energy coupled with the usual kernel smoothing for the density. This results in a variationally consistent SPH scheme in which momentum preservation can be addressed properly.

In problems involving flow through pipes and chokes where solid walls are present, the accuracy of the calculations is sensitive to the treatment of the interaction between the fluid and the solid surface. For simulations of a viscous fluid flow in presence of solid walls, it is necessary to impose no-slip boundary conditions to mimic the sticking of the fluid to the wall. We model such boundary conditions using image particle methods, which in turn are useful in removing the severe crippling density deficiency that arises when the usual SPH continuity equation is applied to particles near a boundary. Image particles are created by reflecting fluid particles across the boundary. This operation results in a collection of imaginary particles which are external to the fluid domain. Such particles are treated as actual SPH particles and so they contribute to the density and pressure gradients. In practice, accurate results for the kernel smoothing of the density are obtained by reflecting no more than four fluid particles aligned in the normal direction to the surface. Unlike actual fluid particles, imaginary particles are not allowed to move relative to the solid surface. Although a velocity is necessarily assigned to each of them, they are always constrained to remain anchored to the solid wall in the course of the calculation. That way, each imaginary particle is given a density equal to the value of its closest image within the fluid domain.

We have performed exploratory two-dimensional model calculations of single-phase flow through a wellhead choke of dimensions like those installed in real production tubing using the Smoothed Particle Hydrodynamics (SPH) method. As a first approximation, we have assumed that negligible heat transfer occurs between different parts of the fluid and so the models were carried out using an isothermal equation of state. The choice of the geometry and parameters are such that the models are well suited to describe the flow of gas through a pipe with a choke throat in the middle as shown in Fig. 1. Three different models were considered which differed either in the form of the outlet boundary condition or the value of the constant coefficient of kinematic viscosity. In model A, the pipe is designed by placing a lid with a small orifice at its end in order to inhibit the outlet flow, while models B and C, differing only in the value of the kinematic viscosity, allow for an infinitely long tube downstream. In all cases, the inlet conditions correspond to Poiseuille flow with a constant density. The results for these model calculations indicate that the flow achieves an approximate steady state in a very short timescale. The stationary solution is always characterized by a well-pronounced drop in the density and pressure through the choke throat. In Fig. 2 we shown the evolution of the density and velocity for model B that corresponds to a calculation with a constant coefficient of kinematic viscosity equal to $\nu = 5.0x10^{-4} cm^2 s^{-1}$.

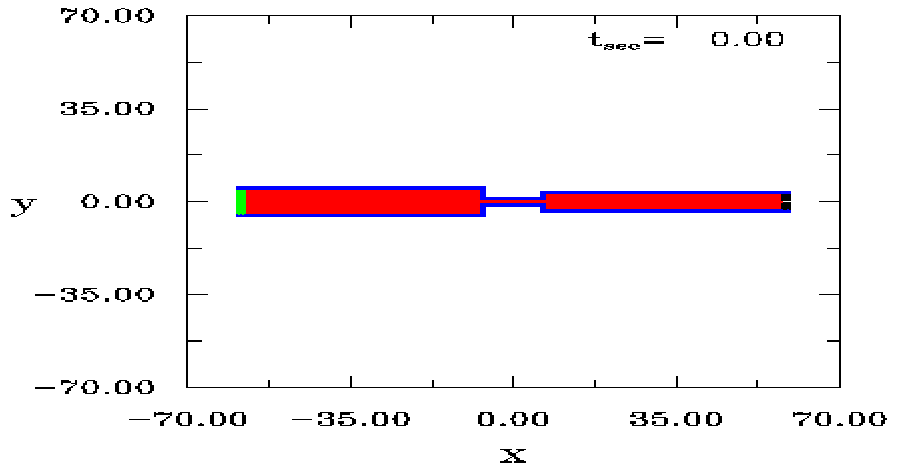

FIG. 1: Geometry of the wellhead choke model used in the calculations. The flow within the choke system is along the x-axis in the direction of increasing x. The numbers on the box sides are given in centimeters

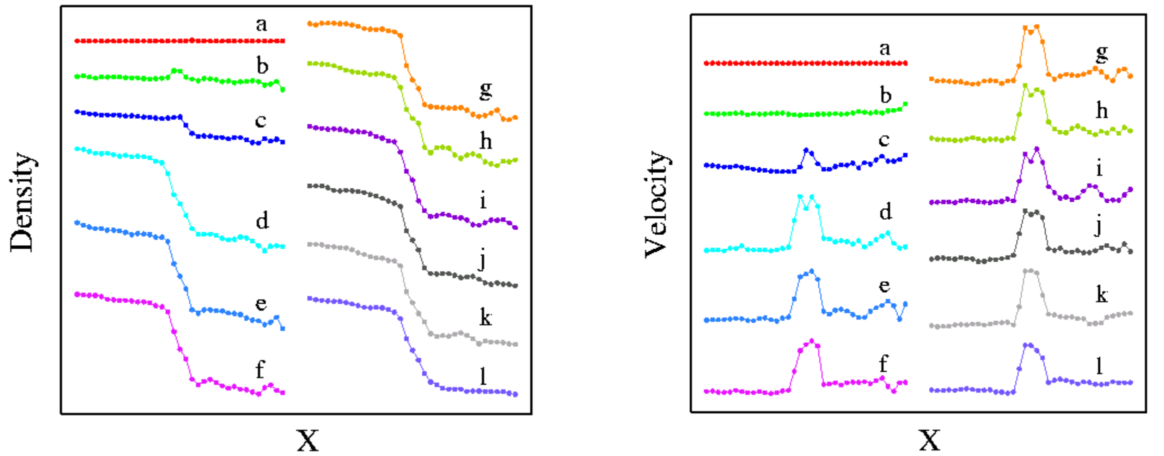

FIG. 2: Mean density (left panel) and velocity (right panel) profiles across the full length of the tube for model B. The curve labels correspond to the following times: 0.0 s (a), 0.0025 s (b), 0.010 s (c), 0.0275 s (d), 0.070 s (e), 0.0875 s (f), 0.1175 s (g), 0.1925 s (h), 0.2025 s (i), 0.230 s (j), 0.270 s (k), and 0.2975 s (l). An approximate steady-state solution is achieved in (d).

At the entrance of the choke the flow velocity increases steeply, reaching typical values which are on average factors of 5 to 6 times higher than those for the upstream flow. The flow velocity also decreases steeply at the choke exit, dropping to downstream values that are slightly higher compared to those for flow in the upstream section of the tube. The flow across the choke throat remains always subsonic with typical velocities of about 0.1c, where c denotes the sound speed. In particular, when the flow is inhibited downstream, the density drop is of about 20% and decreases to 13% or less when the entire outlet cross-section of the tube is free of any restrictions. Because of the isothermal equation of state, the density drop across the choke is equivalent to a pressure drop. Experimental available measurements and correlations for natural gas flowing across wellhead chokes indicate that for speeds of 0.1c, the ratios of the pressure before to that after the choke may be as high as 0.97 - 0.99, implying a much lower pressure drop than predicted by the present calculations. A direct comparison of the results with the available experimental data will certainly be possible for more realistic equations of state.

The present two-dimensional calculations represent a step ahead in consistently simulating choked flow systems. In addition to improving the equation of state, future extensions of this work will include full three-dimensional simulations of flow through wellhead chokes of circular cross-section for both pure gas and gas-liquid mixtures.

References

[1] Hernquist L., Katz N.: TREESPH -- A unification of SPH with the hierarchical tree method. *Astrophys. J.* **70**, 419, 1989.