RELEVANCE OF ALFVEN WAVES IN PROCESS METALLURGY UNDER A HIGH MAGNETIC FIELD

René Moreau*, Kazuhiko Iwai**, Takashi Kameyama**
*Lab. EPM-MADYLAM, ENSHMG, BP 95, 38402 Saint Martin d'Hères Cedex, France
**Department of Materials Processing Engineering, Nagoya University, Furo-cho, Chikusa-ku, Nagoya, Japan 464-8603

Summary Waves are electrically excited at the top free surface and detected at the bottom of a vertical cylinder of liquid gallium, which is located within a vertical superconducting coil generating a 10 Tesla magnetic field. It is shown that, if the frequency is well selected, the pressure disturbance associated to these waves can be detected at the bottom of the apparatus and has all characters of Alfven waves.

INTRODUCTION

It is very common in liquid metal MHD to consider Alfven waves as irrelevant, since their damping time $\mu\sigma L^2$, is usually extremely short (of the order of $10^{-4}\ s$ if the transit length L is $10\ cm$) in comparison with their transit time L/A, where $A = B/\sqrt{\mu\rho}$ is the Alfven celerity. But, with the strong magnetic field delivered by superconducting magnets which now become available, their transit time becomes as short as the damping time, so that the Lundquist number becomes of the order of unity or larger. In this paper, we present a preliminary experiment, performed with liquid gallium at moderate temperature, whose scope is precisely to demonstrate the relevance of these waves. The main characters of such waves, which are derived from an elementary linear theory, are also compared with the measurements. The fairly good agreement between the theoretical predictions and the measurements suggests that this experiment may be considered as a demonstration of the relevance of Alfven waves in liquid metals under high magnetic fields.

THE EXPERIMENT

A $12\ cm$ column of liquid gallium is located in a vertical cylinder made of an electrically insulating material and this vessel is itself centered within a vertical coil, which may generate a $10\ Tesla$ DC magnetic field. A pressure sensor is inserted from below at about $2\ cm$ above the bottom wall and is connected to a noise filter and an oscilloscope. At the top free surface, two cupper electrodes are partially immersed in the liquid metal. They are connected to a power supply capable to generate an AC electric current whose frequency is fixed to $1\ 500\ Hz$ and whose intensity may vary from zero to $90\ Amp$. In the presence of both the vertical magnetic field and the horizontal current passing between the electrodes, the fluid is locally submitted to an oscillating horizontal force, which is itself partially balanced by a pressure force. This results in an unsteady fluid motion, which must be organized as a set of vortices, which propagate and form a traveling wave. According to Ohm's law, associated to the fluid motion, there exists also a set of induced electric current loops. The energy of each vortex and each current loop depends on its distance from the excitation area. With a moderate magnetic field, one would expect that the energy would be damped out rapidly, essentially by Joule effect, since viscosity is quite small whereas the magnetic diffusivity is 10^7 larger, so that this column of vortices would not reach the bottom wall. But, in the presence of the strong DC magnetic field, since the Alfven waves propagate very fast, it may be expected that a significant fraction of this energy should reach the bottom wall. Indeed, the condition to satisfy in order to observe this behavior is to have a Lundquist number $\mu\sigma AL$ at least of the order of unity. This is precisely what we get with a magnetic field of $10\ Tesla$.

The main experimental results, which will be presented, are pressure fluctuations delivered by the pressure sensor. They exhibit the same frequency as the electric current supplied to the electrodes. A spectral analysis of this signal also reveals a second peak at twice that frequency. And it is clearly shown that the height of this second peak varies linearly with the square of the intensity supplied to the electrodes, whereas the height of the first peak varies proportionally to that intensity. This is a specific property of Alfven waves.

A LINEAR THEORY

We start from the usual equations of MHD, and we neglect viscosity since the magnetic Prandtl number of liquid metal is extremely small, so that the ohmic damping is the only remaining dissipation mechanism. We also neglect the non-linear part of inertia in the Navier-Stokes equation, because the energy of the system remains small. The magnetic field is expressed as $\mathbf{B}=B_0\,\mathbf{i}_z+\mathbf{b}$, $|\mathbf{b}|<<B_0$, where the magnetic field disturbance $\mathbf{b}$ and the fluid velocity are proportional to $exp[i(\mathbf{k}\cdot\mathbf{r}-\omega t)]$. And the wave vector is supposed to be of the form $\mathbf{k}=(k_\perp,0,p+iq)$, where the three quantities $k_\perp, p, q$ are real. This means that the cylindrical vessel is modeled as a 2D rectangular domain. And q represents the attenuation of the wave along the distance from the excitation area, whereas $k_\perp, p$ represent the shape of the vortices in the rectangular plane. The linearized equations obviously lead to a dispersion equation whose analysis yields the typical properties of the waves.

The main theoretical results may be expressed in terms of the Elssaser number $E=\mu\sigma A^2/\omega=\sigma B^2/\rho\omega$ built with the two main data: the applied magnetic field and the frequency of the excitation. They allow to analyze the conditions in which the waves persist all over the domain, in spite of their important and rapid damping. Essentially, it is shown that there exists a range for this number in which the waves are observable without a too strong damping, it is roughly between the two limits 1 and $1/M^2$, where $M=k_\perp/\mu\sigma A$. The smaller value corresponds to the limit below which the skin effect is significant and the excited disturbance does not escape from the skin depth. And the larger value corresponds to a shape factor such that the eddies and the current loops are squeezed between vertical walls too close from each other to allow the development of the induced current without important ohmic losses. Finally, the most efficient value for the Elssaser number appears to be around $\mu\sigma A l_\perp$, where $l_\perp\approx 1/k_\perp$ is the horizontal dimension of the rectangular domain. A numerical application to the case of a gallium experiment suggests a discrete set of frequencies. Each of them corresponds to an entire number of half vortices. And, among them, the frequency $1430\ Hz$, which is remarkably close of the experimental value $1500\ Hz$, can be associated with a flow pattern made of 5 half vortices between the free surface and the bottom wall. In this case, the amplitude of the velocity of the fifth vortex is still 2/3 of that of the vortex present at the free surface.

CONCLUDING REMARKS

The main theoretical conclusion of this work is a prediction of the best conditions to observe Alfven waves in a rectangular domain filled with a liquid metal. It may be illustrated in a quarter plane representing the applied magnetic field versus the frequency of the excitation. The domain where the waves persist is a fraction of this quarter plane, between two curves. One of them gives a minimum magnetic field, such that $E>1$, the other gives a minimum frequency related to the skin effect. Those conditions can easily been applied to metals other than gallium, such as steel or other metallic alloys. In all cases, with the usual physical properties of liquid metals, the minimum magnetic field should be above a critical value of the order of $2\ Tesla$.

And the main conclusion of the experiment, beside the demonstration that the Alfven waves can actually be observed in liquid metals, is indeed a suggestion to repeat and to complement those measurements, with higher magnetic filed and with more sophisticated diagnostic techniques.