XXI ICTAM, 15–21 August 2004, Warsaw, Poland

# WEAK-TURBULENT THEORY OF WIND-DRIVEN SEA

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*Summary* We developed a weak turbulent theory of the wind-driven sea, based on the assumption that the main physical process is the resonant nonlinear four-wave interaction. This model far developed analytically, supported by the massive numerical experiments is possible to explain from the first principles the balk of experimental facts on the wind-driven surface wave turbulence collected in physical oceanography.

Physical oceanography collected a huge amount of experimental data on the wind-driven sea. These data include frequency and angular-frequency spectra of surface elevation as well as spatial spectra measured from planes and satellites. A lot of collected data present fetch and duration dependance of such sea integral characteristics as energy $\epsilon =< \eta^2 >$ and the peak frequency $\omega_p$. This variety of experimental data can be essentially reduced by the use of Kitaigorodskii similarity conjecture [1], stating that the major features of wind-driven sea can be expressed in terms of dimensionless variables (here $U$ is wind velocity):

$$\nu = \frac{\omega_p U}{g}, \qquad e = \frac{\epsilon g^2}{U^4}, \qquad \chi = \frac{xg}{U^2}.$$

Experiments show that $e, \nu$ are powerlike functions on the dimensionless fetch $\chi$:

$$e = u\chi^p, \qquad \nu = v\chi^{-q} \tag{1}$$

Here $0.75 < p < 1.0$, $0.24 < q < 0.3$ and $u = (5 \sim 10) \cdot 10^{-7}$, $v \simeq 10$ are constants, which vary in different studies. Omnidirectional frequency spectra can be presented in the universal form:

$$F(\omega) = \frac{g^2}{\omega^5} F\left(\frac{\omega}{\omega_p}, \nu\right)$$

The dependance on $\nu = 1/c$ ($c$ is a "wave age") is relatively slow.
Since the seminal works of Hasselmann [2], it is widely accepted that the wind-driven sea can be described by the kinetic wave equation imposed to the spatial spectrum of wave action $N(\vec{k})$:

$$\frac{\partial N_k}{\partial t} + \frac{\partial \omega}{\partial \vec{k}} \frac{\partial N_k}{\partial \vec{r}} = S_{nl} + S_{in} + S_{ds} \tag{2}$$

Here $S_{in}$ is the wind input, $S_{ds}$ is dissipation term, and $S_{nl}$ is nonlinear transfer term describing exchange of wave energy due to four-wave resonant nonlinear processes. $S_{nl}$ can be calculated exactly from the Euler equation for potential flow of the fluid with free surface in presence of gravity. The term $S_{in}$ cannot be found analytically due to a strong turbulence in the atmospheric boundary layer. The excitation of waves by wind is the Cherenkov-type instability. It leads to the following phenomenological form of $S_{in}$:

$$S_{in} = \beta \cdot N_k, \qquad \beta \simeq \mu \frac{\rho_a}{\rho_w} \omega f(\xi), \qquad \xi = \frac{U\omega \cos\theta}{g}$$

Here $f(\xi) = 0$ if $\xi < 1$, and $\mu \simeq 0.1$ is an empiric small parameter, appearing due to turbulence of atmosphere, $\rho_a, \rho_\omega$ are densities of air and water. For $\xi \gg 1$, we have $F(\xi) \simeq \xi^\kappa, 1 < \kappa < 2$. The term $S_{in}$ is known from the experiments with poor accuracy (scatter of $S_{in}$ has the same order as $S_{in}$).
The most important mechanism of wave energy dissipation is generation of parasitic capillary harmonics on the breaking wave crests. This process takes place uniformly on the whole ocean surface. At high wind velocities it turns to the "microbreaking" process. This process leads to absorption of energy in the high-frequency part of wave spectra. The direct white-capping on the crests of leading waves is the dissipation mechanism of secondary importance, responsible for slow saturation of the downshift process for waves moving faster than wind. There are neither analytical nor serious experimental studies making possible to derive a well-justified expression for $S_{ds}$. Some hand-woven forms of $S_{ds}$ are used worldwide in the operational models of wave prediction (WAM, SWAN, *etc.*) Mostly they are constructed in a way to provide the local in frequency energy balance, i.e.

$$S_{in} + S_{ds} = 0$$

In these scenario $S_{nl}$ terms plays an auxiliary role. In our report we offer a completely different approach [3-6]. In accordance with all summary of experimental facts we assume that $S_{ds}$ is essential only in the region of high wave numbers, where it plays the role of a "universal sink" and does not affect essentially spectral dynamics in the area of spectral peak. In the leading order the wind-driven sea can be described by the conservative kinetic equation

$$\frac{\partial N_k}{\partial t} + \frac{\partial \omega}{\partial \vec{k}} \frac{\partial N_k}{\partial \vec{r}} = S_{nl} \tag{3}$$

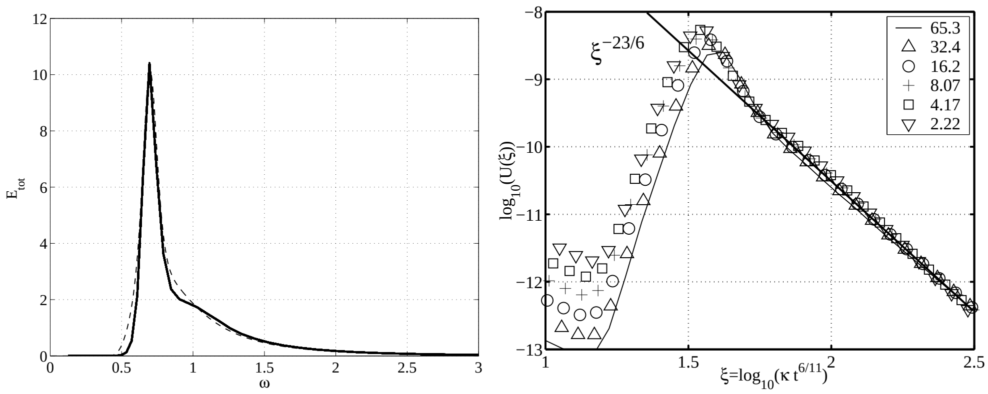


**Figure 1.** Left panel — comparison of numerical solutions (hard line) for the kinetic equation with JONSWAP approximation (dash) for wave age 1.42 (time 5.3 hours). Generation is given by Hsiao & Shemdin (1983) formula, wave speed is 20m/s. Dependence of $\alpha$ on wave age follows Babanin & Soloviev (1998) with the exponent $\kappa = 1$ and $\alpha_0 = 0.00127$. Right — self-similar function $U(\xi)$ ($\xi = |\mathbf{k}|t^{6/11}$) for different times (see legend, in hours). Bold line is the Kolmogorov inverse cascade solution $U \sim \xi^{-23/6}$. Wave input is given by Hsiao & Shemdin formula for wind speed 10 m/sec

together with the wave-action balance equation

$$\left\langle \frac{\partial N_k}{\partial t} \right\rangle + \left\langle \frac{\partial \omega}{\partial \vec{k}} \frac{\partial N_k}{\partial \vec{r}} \right\rangle = \langle S_{nl} \rangle \tag{4}$$

The brackets presume integration over the $\vec{k}$-space. Eq.(2) describes weak turbulence that is governed by the direct cascade of energy and the inverse cascade of wave action. Both uniform

$$\frac{\partial N_k}{\partial t} = S_{nl} \tag{5}$$

and stationary

$$\frac{\partial \omega}{\partial \vec{k}} \frac{\partial N_k}{\partial \vec{r}} = S_{nl} \tag{6}$$

versions of Eq.(1) have two-parameter families of self-similar solutions. The parameters could be found from balance Eq.(2). The self-similar solution of Eq.(6) in dimensionless variables reads:

$$n(\kappa, \chi, \theta) = b^5 \chi^{5\beta - 1/2} P_\beta \left( b\chi^\beta \kappa, \theta \right), \tag{7}$$

where $b, \beta$ are free constants. Eq.(6) presumes that for Eq.(1)

$$q = \frac{2p+1}{10}, \qquad v = u^{1/5} C_\beta, \qquad C_\beta \simeq 1 \tag{8}$$

Relations (8) are very well confirmed by experimental data. Eq. (5) has following self-similar solutions

$$n(\kappa, \tau, \theta) = b^{19/4} \tau^{\frac{19\beta - 2}{4}} P_\beta (b\kappa\tau^\beta, \theta) \tag{9}$$

Here $\tau = tg/u$ is a dimensionless duration. Solution (9) is in accordance with the experimental data that in the case of duration-limited studies are scant. However for Eq. (5), asymptotic self-similarity of its solution is confirmed very well by massive numerical experiments [4,6]. It is remarkable that the shape of frequency spectra, obtained in these computations, almost perfectly fits the experimental data collected in the fetch-limited studies (Fig.1).

## References


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