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CONTINUUM THERMODYNAMIC AND VARIATIONAL MODELING AND SIMULATION OF DUCTILE FAILURE AT LARGE DEFORMATION WITH APPLICATION TO ENGINEERING STRUCTURES

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Summary
The purpose of this work is the extension of local Gurson-based ductile damage and failure modeling in engineering materials and structures to account for the non-local nature of void coalescence. In particular, the extension pursued here is based on the introduction of an non-local effective damage parameter ν analogous to that f^* of Needleman and Tvergaard which is modeled thermodynamically as a scalar-valued continuum microstructural field or generalized phase field via a recent thermodynamic approach. In the simplest case, the resulting field relation for ν is formally analogous to the inhomogeneous temperature equation. As such, analogous to temperature, ν represents an additional continuum degree-of-freedom here. And in the complete model, damage and deformation are coupled. Further, the field relation for ν contains a characteristic length determining the effective dimension of the process zone for void coelescence.

INTRODUCTION

The modeling of fracture in ductile metals due to damage is often based on the micromechanical model of [1] for the growth of a single void in an ideal elastoplastic matrix. In order to account for the effects of void nucleation and coalescence, and so obtain better agreement between the model, experimental results and numerical simulations for ductile failure and crack propagation, the original Gurson model, and in particular the Gurson yield function, was modified and extended into a semi-phenomenological form by [2], [3], and [4]. In the ensuing years, this modified approach has been extended yet further in a hypoelastic-based incremental fashion (*e.g.*, [5]) to account for isotropic and kinematic hardening, as well as for large inelastic deformation. In addition, a hyperelastic-based approach ([6], [7]) to such modeling has been used successfully to model ductile crack propagation in fracture mechanics specimens subject to more complex, and in particular to cyclic, loading histories. More recently, Gurson-based modeling has been extended to non-local form (*e.g.*, [8], [9]) in order to deal with mesh-dependence, and to model void coalescence as a non-local process ([10], [11]). In particular, the latter approach results in a delocalization of the model damage process and a minimization of mesh-dependence, such that ductile failure takes place in a region rather than at a point. In comparison to the local modeling, the proposed non-local extension involves a single additional material parameter, *i.e.*, that involving the characteristic lengthscale. The current extension reduces in fact to the existing local Gurson-based modeling as this lengthscale is reduced to zero.

EXTENDED GURSON-BASED DUCTILE DAMAGE MODELING

The non-local model formulation from [10] and [11] is based on the introduction of an effective continuum damage field ν (formally analogous to that f^* introduced by [3]) into the Gurson yield function. This damage parameter is itself modeled as a continuum microstructural field or generalized phase field via a recent thermodynamic approach ([12], [13]). The incorporation of this field into the existing Gurson-based framework is based on the assumption that the effect of ν on the yield behaviour due to ductile damage is the same as that of f^* on this behaviour in the local Gurson-based modeling of [3]. On this basis, ν is incorporated into the current model via the form

$$\phi = \frac{\sigma_{\mathrm{V}}^2(\boldsymbol{T})}{\sigma_{\mathrm{M}}^2} - 1 + 2\,q_1\,\nu\,\cosh\!\left(q_2\,\frac{\boldsymbol{I}\cdot\boldsymbol{T}}{2\,\sigma_{\mathrm{M}}}\right) - q_1^2\nu^2 \tag{1}$$

for the yield function ϕ formally analogous to the Gurson form. Here, $\sigma_{\mathrm{V}}(\boldsymbol{T})$ represent the von Mises stress as based on the Cauchy stress $\boldsymbol{T}$, σ_{M} the yield stress of the matrix material. Futher, q_1 and q_2 are constants. Again, note that this extension bears a formal resemblance to that formulated by [3] and embodied in their effective damage parameter f^*, which they introduced in order to account for the effect of void coalescence on the yield behaviour. Representing a quantity which characterises in an effective fashion the transition from the undamaged to the damage state, the field ν is modeled here as a continuum microstructural field or generalized phase field via a recent thermodynamic approach ([12], [13]) to the modeling of such structure. Specializing this general approach to the case of quasi-static conditions and a single scalar-valued microstructural field representing phenomenological damage, one derives in particular the basic evolution relation

$$\mathrm{div}(d_{,\nabla\dot{\nu}}) - \psi_{,\nu} - d_{,\dot{\nu}} = 0 \tag{2}$$

for the continuum damage field ν in terms of the forms

$$\begin{aligned} \psi &= \psi(\ln\boldsymbol{V}_{\mathrm{E}},\nu)\,, \\ d &= \alpha_{\mathrm{D}}\,(\dot{\nu}-\dot{f}^*)^2 + \tfrac{1}{2}\,\alpha_{\mathrm{D}}\,\ell_{\mathrm{D}}^2\,\nabla\dot{\nu}\cdot\nabla\dot{\nu}\,, \end{aligned} \tag{3}$$

for the free energy density ψ and dissipation potential d, respectively. Here, $\ln \boldsymbol{V}_{\mathrm{E}}$ represents the elastic logarithmic left stretch tensor determining the Cauchy stress $\boldsymbol{T}$ via

$$\boldsymbol{T} = \det(\boldsymbol{F})^{-1}\psi_{,\ln \boldsymbol{V}_{\mathrm{E}}} \quad . \tag{4}$$

In addition, α_{D} represents a characteristic energy scale, ℓ_{D} a characteristic lengthscale, and τ_{D} a characteristic timescale, for the non-local damage process. Among other things, the con stitutive form for d is based on the idea that the nucleation, growth and coalescence of voids in the guise of $\dot{f}^{*}$ represents the source of continuum damage here.
Using the discretization of the weak field relations, the algor ithmic form of the model relations were implemented in the finite element programm ABAQUS/Standard using the user-inter face UEL. In addition, the local version of the model was implemented into ABAQUS/Standard using the user-interface UM AT.

MODELING AND SIMULATION OF DUCTILE CRACK PROPAGATION IN A C(T) SPECIMEN

In this section, the behaviour of the above gradient-type ex tension of Gurson-based ductile damage modeling is compared with the corresponding local modeling in the context of the si mulation of large mode 1 ductile crack growth in a C(T) specimen (Figure 1). Using material parameter values typica l for the ductile steel 10MnMoNi5-5 at room temperature, such crack growth was modeled using the local and non-local fo rmulations.

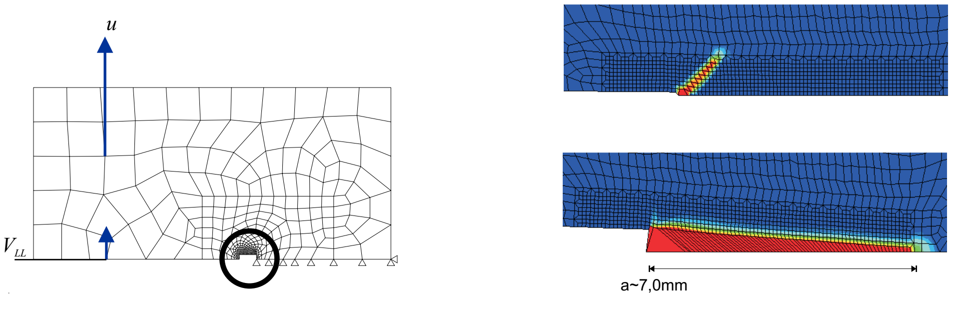

Figure 1. Simulation of mode 1 ductile crack propagation in a C(T) spec imen (left) using the local (upper right) and non-local (lower right) models.

Figure 1 (left) displays the finite element discretization o f the upper half the of the compact tension specimen. In parti cular, in the fracture zone, a regular element distribution with an e lement side length of 0.05 mm was used. Simulation results based on the local model (upper right) and non-local model ex tension (lower right) for the region around the crack tip (circled) are shown. As shown by the ellipticity and stability a nalyses of the local model, loss of ellipticity of the local model results in a bifurcation at crack initiation and an unr ealistic crack-path prediction that is extremely mesh-dep endent. On the other hand, the use of the non-local damage model predic ts stable mode 1 crack growth as observed in experiment.

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