MICROMECHANICAL STUDY OF MACROSCOPIC FRICTION AND DISSIPATION IN IDEALISED GRANULAR MATERIALS: THE EFFECT OF INTERPARTICLE FRICTION

N.P. Kruyt*, L. Rothenburg**
*Department of Mechanical Engineering, University of Twente, Enschede, The Netherlands
**Department of Civil Engineering, University of Waterloo, Waterloo, Ontario, Canada

Summary A micromechanical study is made of the relationship between interparticle friction coefficient μ and macroscopic continuum friction and dissipation in idealised granular materials, using Discrete Element Method simulations with varying μ. As expected, macroscopic friction and dilatancy increase with μ. Surprisingly, dissipation is present even when $\mu \to 0$ or when $\mu \to \infty$. Hence, dissipation in idealised granular materials is not exclusively the result of interparticle friction. The dependence of the dissipation-rate function on plastic strains is also investigated.

INTRODUCTION

Granular materials exhibit macroscopic frictional and dissipative behaviour, which usually is modelled by the theory of plasticity. Currently, there is considerable interest in thermodynamic formulations of plasticity theories for geomaterials, e.g. [2]. In these formulations, knowledge of the (Helmholtz) free-energy function and the dissipation-rate function is sufficient for determining the constitutive behaviour, such as elastic behaviour, yield function, flow rule and hardening rule. The macroscopic frictional and dissipative nature of granular materials seems to be readily explained by the frictional interaction between particles at the microscopic contact level. However, experimental evidence [8] and computational evidence [1] suggests that the macroscopic frictional behaviour is only weakly dependent on interparticle friction coefficient μ (except for very low μ).

The objective of this study is to investigate further the micromechanical origin of macroscopic friction and dissipation in granular materials, and the role of interparticle friction herein. The method used is that of Discrete Element Method (*DEM* for short) simulations on idealised two-dimensional granular assemblies consisting of disks. A great advantage of DEM simulations for micromechanical studies is that detailed information is available, such as that on interparticle forces. Thus it is possible to compute for each state the internal energy associated with the springs that are active between particles in contact.

In particular, the following cases are considered: (i) $\mu \to 0$, (ii) 'normal' interparticle friction coefficients and (iii) $\mu \to \infty$. The first case corresponds to frictionless particles with central interactions for disks (no tangential forces), while in the third case the Coulomb friction limit is never attained. In the first and third cases the behaviour at the contact is elastic (as long as the contact is not disrupted) and hence there is no microscopic frictional dissipation mechanism.

DISCRETE ELEMENT SIMULATIONS

Two-dimensional DEM simulations were performed in a manner similar to that of [3]. A linear elastic relationship between forces and interparticle displacements at contacts has been employed, i.e. $f_n^c = k_n \Delta_n^c$ and $f_t^c = k_t \Delta_t^c$, where f_n^c, f_t^c are normal and tangential contact forces, Δ_n^c, Δ_t^c are corresponding interparticle displacements and k_n, k_t are respective stiffnesses. Only compressive normal forces are allowed: if the normal force were to become tensile, the contact is considered to be broken for cohesionless materials. The tangential forces are limited by Coulomb friction, i.e. $|f_t^c| \le \mu f_n^c$.

Simulated biaxial tests employing periodical boundary conditions were performed on assemblies of 50,000 disks. By simulating additional unloading paths, plastic strains were determined.

EFFECT OF INTERPARTICLE FRICTION COEFFICIENT ON MACROSCOPIC BEHAVIOUR

The results for the macroscopic shear strength and the volumetric strain are shown in Figure 1 for various values of μ. In all cases the shear strength shows behaviour typical for a dense granular material with an initial increase of shear strength until the peak strength is obtained after which softening occurs. In all cases, shear-induced dilation is observed. The volumetric strain and the shear strength (at peak and at large strain) increase with μ. As noted in [1], the effect of μ is fairly small in the range $0.2 - 0.4$. Thus, the qualitative behaviour for $\mu \to 0$ and $\mu \to \infty$ is the same as that for the cases with a normal friction coefficient. This means that the macroscopic behaviour is frictional in nature, even when friction is effectively absent at the microscopic contact level.

DISSIPATION

The dissipation-rate function, $\hat{\Phi}$, is related to the rate at which work is done on the assembly, $\sigma_{ij}\dot{\varepsilon}_{ij}$, and the rate of change of the free-energy function, $\dot{F}$, by $\hat{\Phi} = \sigma_{ij}\dot{\varepsilon}_{ij} - \dot{F}$ for isothermal systems [2]. Contrary to the internal energy u, the free energy can not be determined from the DEM simulations. Here we make the assumption $u \cong F$, as was also done in an experimental study of dissipation in metals [6]. Then the dissipation-rate $\hat{\Phi}$ can be determined.

Surprisingly, dissipation is even present when $\mu \to 0$ or when $\mu \to \infty$. Thus there is macroscopic dissipation in the absence

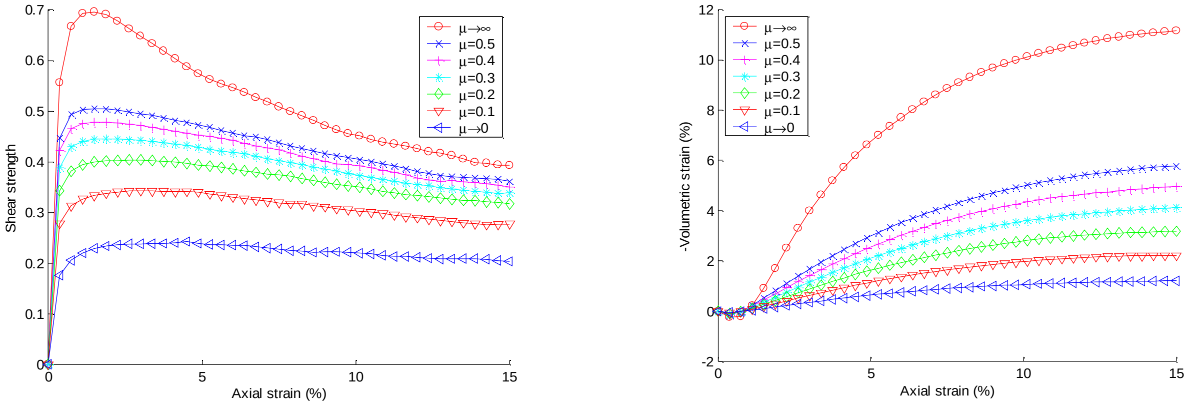

Figure 1. Macroscopic behaviour as a function of imposed axial strain for various interparticle friction coefficients μ .

of a microscopic frictional dissipation mechanism. Ultimately, another microscopic mechanism is always present in DEM simulations as viscous or numerical dissipation. The first is often modelled by a coefficient of restitution. Some dissipation mechanism is necessary to dampen out (uninteresting) transient phenomena associated with elastic vibrational modes.
In [2] a distinction is made between recoverable plastic work and dissipated plastic work. For a ‘purely frictional material’ all plastic work is dissipated. Since simulations of unloading paths have been performed, plastic strains ε_{ij}^{p} are known. Hence the rate of plastic work, $\sigma_{ij}\dot{\varepsilon}_{ij}^{p}$, can be determined and compared with the dissipation-rate $\hat{\Phi}$. It is found that for axial strains larger than 3% (5% for $\mu \to \infty$) these are practically the same. Thus the present, idealised granular material is a ‘purely frictional material’ where none of the plastic work is recoverable.
The dependence of the dissipation-rate function on plastic strains was also investigated. A form proposed in [7] is: $\hat{\Phi} = N(\mu) p a_c \dot{\varepsilon}_S^{p}$, where p is the pressure, a_c is the anisotropy of the fabric tensor (or contact distribution function), $\dot{\varepsilon}_S^{p}$ is the rate of change of the magnitude of the deviator of the plastic shear strain and $N(\mu)$ depends solely on μ . The actual and the theoretical dissipation-rate functions are compared in Figure 2, with $N(\mu)$ determined from steady-state considerations. The theoretical relation underestimates the dissipation as observed in the DEM simulations.

DISCUSSION

It has been shown that, even in the effective absence of microscopic friction ($\mu \to 0$ or $\mu \to \infty$), the macroscopic behaviour is frictional and dissipative. The role of microscopic interparticle friction in influencing the macroscopic behaviour is more indirect by giving stability to assemblies, thus leading to increased shear strength and dilatancy with increasing μ .
Further attention must be given to ways in which: (i) the free energy and (ii) the recoverable plastic work can be extracted from the results of DEM simulations. According to [5], for metals this recoverable plastic work is associated with microscopic heterogeneity. Granular materials are very heterogeneous at the microscopic level of contacts, e.g. [4]. Yet, the present results indicate that the recoverable plastic work is small.

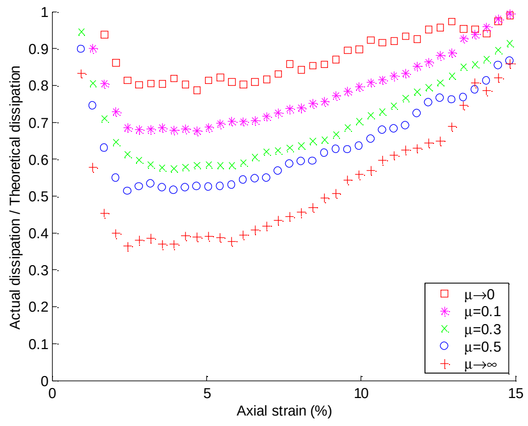

Figure 2. Comparison of dissipation-rate functions.

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