An idea and theory of hypothetical device for investigating the localization phenomena

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Summary Suggested and supported by the stability analysis is an idea of the lamination shear device with a domain of deformation of rather arbitrary shape. The device is intended to clarify the relation between the localization pattern and the shape of solid deformed.

The localization phenomena considered as a mechanism of incipiency of discontinuities in solids, are of great interest for a number of theoretical and applied disciplines. Systems of discontinuities that arise from the zones of localized deformation, inherit the geometry of corresponding localization patterns. The ralation between the localization pattern, that arises as a pecular mode of instability of a uniform or quasiuniform deformation, and the shape of doma in of deformation, attracts attention of the researchers for a long period, still remaining unclear.

For experimental study of the relation pointed out the devices are needed that would provide the following: (1) uniformity of the basic deformation process; (2) the possibility to set different (ideally, arbitrary) shapes of the deformation domain; (3) stability of the uniform deformation up to violation of strong ellipticity, which stipulates the onset of localized instability.

In this work an essential scheme of such a device is suggested and it is proved theoretically that the properties listed above do take place.

The device itself is a pile of thin laminae (fig.) in which a cavity of certain shape is cut. The lamination forms a parallelepiped confined from two opposite sides by two rigid and smooth flat walls (not represented on the figure), parallel to each other; corresponding side faces of the lamination can slide over inner surface of the walls. The cavity reaches one or both walls which close its side openings. It is filled by a homogeneous material tested (bulk or soft plastic one), which forms the body subsequently called the "specimen". The lamination undergoes prescribed quasistatic simple shearing in the plane paralled to the walls. Then a possible deformation of the specimen is the same uniform simple shearing, which is considered as the basic deformation process (note that certain part of the specimen boundary contacts the walls, with sliding being assumed there). If the basic process is stable, it is unique and hence its uniformity will be preserved up to the onset of instability.

Let us show that under rather loose assumptions regarding the material tested, the specimen stability is preserved as long as the incremental constitutive relation remains strongly elliptic. We suppose that the material is initially specularly symmetric with respect to the plane of shearing (parallel to the smooth walls); in particular, it can be initially isotropic or orthotropic. In that case it can be proved readily that during the uniform shearing not only homogeneity of the material is preserved, but additionally the plane of its specular symmetry, which remains parallel to the planes of boundary sliding. Disturbing infinitesimal displacements (additional to those of the basic process) vanish over the part of the specimen boundary that lies inside the lamination and are tangential over its flat parts contacting the smooth side walls and sliding over them. Thus, the material symmetry of the specimen and its kimematical perturbation boundary conditions correspond to the hypothesis of a modification (one of those proved by the author [1, 2]) of the well–known Van Hove's theorem [3], which enables to prove stability of the basic process up to violation of strong ellipticity. Taking into account, that the stability criterion (going back to studies by D. Drucker and R. Hill) is nowadays standard and commonly known, we state only the abovementioned Modified Van Hove's theorem, making use of which is crucial for the proof of stability.

Modified Van Hove's theorem. Let B be a regular subregion of the plane layer with a boundary $\partial B = \partial B_1 \cup \partial B_2$, ∂B_1 lying inside the layer and ∂B_2 lying on its boundary planes; let a constant fourth-order tensor $\mathbf{C}$ be strongly elliptic and specularly symmetric with respect to the plane parallel to boundary planes of the layer; let vector field $\mathbf{u}(\mathbf{r})$ be continuous, piecewise-smooth, vanishing on ∂B_1, and tangential on ∂B_2. Then the functional $\int_B \nabla \otimes \mathbf{u} : \mathbf{C} : \nabla \otimes \mathbf{u} dV$ is definitely positive.

Since the stability criterion is just definite positiveness of the functional of such a type, we conclude that asserted stability up to the bound stated is proved. Thus, instability of the uniform deformation arises as a result of violation of strong ellipticity and hence is localized. As for the experiments with different shapes of the cavity, they can help to reveal the influence of the shape on geometry of the localization pattern.

Note that the lamination shear device with definite (the parallelepipedal) shape of cavity was described (without any kind of stability analysis) in [4].

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References

[1] *Ryzhak E.I.* On stable deformation of "unstable" materials in a rigid triaxial testing machine. J. Mech. Phys. Solids **41**: 1245-1256, 1993.

[2] *Ryzhak E.I.* On stability of homogeneous elastic bodies under boundary conditions weaker than displacement conditions. Q.J. Mech. Appl. Math. **47**: 663-672, 1994.

[3] *Van Hove L.* Sur l'extension de la condition de Legendre du calcul des variations aux integrales multiples a plusieurs functions inconnues. Proc. Kön. Ned. Akad. Wet. **50**: 18-23, 1947.

[4] *Revuzhenko A.F. et al.* On the mechanism of deformation of a bulk material at large shears. Soviet Mining Science **N 3**: 130-133, 1974.

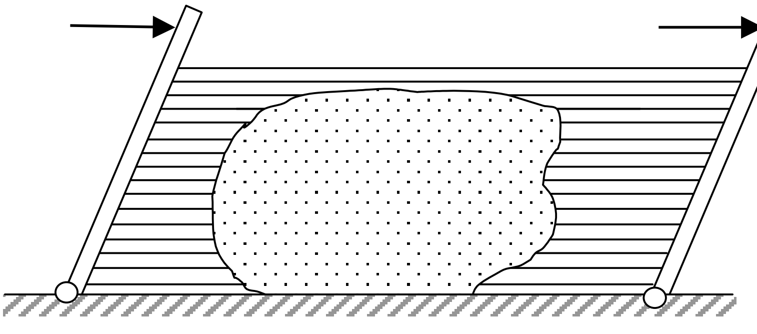

Fig. A plane scheme of the device. The figure plane is parallel to the plane of shearing and to the planes of two smooth side walls (not represented), the latter closing side openings of the cavity